Using visualizations to teach problem-solving skills in mathematics: Which kind of visualization works?

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USING VISUALIZATIONS TO TEACH PROBLEM-SOLVING SKILLS IN MATHEMATICS: WHICH KIND OF VISUALIZATION WORKS?

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Abstract. In the experiment described in this paper we investigated the effects of different kinds of computer-based visualizations on the acquisition of problem-solving skills in the domain of probability theory. Learners received either purely text-based worked examples, text plus an instruction to mentally imagine the examples' contents, or they could retrieve either static pictures or concrete animations that depicted the problem statement and the problem states achieved by applying a specific solution step. It could be shown that frequently using static pictures or imagining the examples' contents both improved problem-solving performance on isomorphic problems. However, there were no positive effects of using animations. Rather, the frequent use of animations led to substantial increases in learning time, while it slightly decreased performance at the same time. Thus, the use of concrete animations to visualize solution procedures was more harming than helpful for conveying problem-solving skills.

1. INTRODUCTION

Understanding mathematical solution procedures is a necessary prerequisite in order to be able not to solve only familiar problems, but also to work on novel problems requiring an adaptation of known solution procedures. However, it has been noted that students often face severe difficulties in understanding solution procedures even when they have received elaborated instructional explanations of the individual solution steps. This may result from the fact that the solution steps are often conveyed in a rather abstract way so that learners experience difficulties in imagining which changes in the problem state are achieved by applying a specific solution step to a problem.

The use of multimedia learning environments may offer ways to overcome these difficulties that can hardly be achieved by traditional instructional means (Mayer, 2001). In multimedia learning environments information presentation can be accomplished by using different representational formats (textual and pictorial) which may be processed in different sensory channels (auditory and visual). Additionally, information presentation is not restricted to static displays (e.g., diagrams, pictures, written text), but the representations used can involve changes over time (e.g., dynamic visualizations, spoken text).

In the current paper we are interested in the effects of augmenting a purely text-based hypertext environment called HYPERCOMB by different kinds of visualizations. HYPERCOMB teaches how to calculate the probability of complex events. Problems of calculating complex event probabilities are related to situations where the probability of selecting a particular configuration of elements randomly out of a set of elements has to be determined (cf. Figure 1).

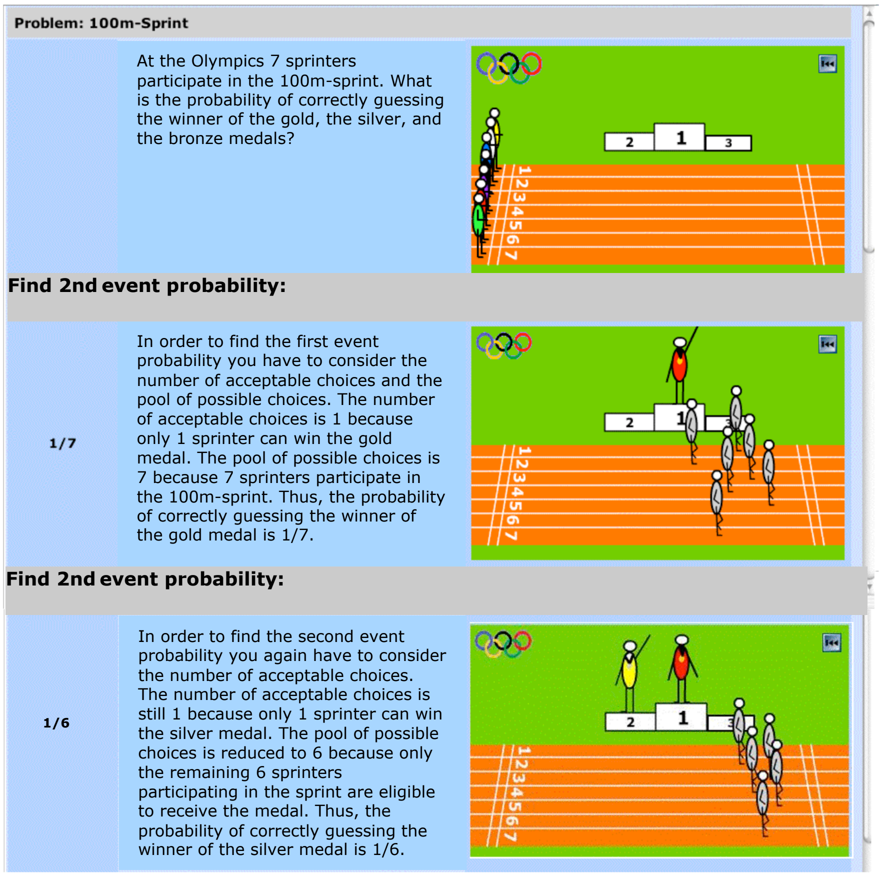

Figure 1: Screenshot of a worked-out example with external visualization

The main instructional principle underlying HYPERCOMB is the use of worked-out examples for conveying knowledge on different problem categories. Research over the last 20 years has shown that worked-out examples are of great help for knowledge acquisition in particular in well-structured domains like mathematics, physics, or programming (Atkinson, Derry, Renkl, & Wortham, 2000). However, although we have identified a way of designing worked-out examples that boosts performance compared to conventionally designed examples (Gerjets, Scheiter, & Catrambone, 2004) there is still space left for improvements. We assumed that this space might be claimed by the benefits achieved through the use of visualizations.

According to the multimedia principle (Mayer, 2001) embellishing textual learning materials by static pictures or dynamic visualizations (i.e., animations) helps to promote learners' understanding of instructions. With regard to the acquisition of problem-solving knowledge visualizations of worked-out examples may first help learners to understand the situation described in the problem statement (i.e., the initial problem state) and thus to correctly represent its meaning in a situation model (Nathan, Kintsch, & Young, 1992). Second, visualizations of the solution steps may promote an understanding of changes with regard to the initial problem state that are achieved by applying a solution step to a problem. Visualizing worked-out examples can be done by presenting either static pictures, animations, or an instruction to mentally imagine the examples' contents.

Static Pictures. Static pictorial representations are known to foster the immediate and delayed retention of facts contained in the accompanying text (cf. for a review Levin, Anglin, & Carney, 1987). Moreover, with regard to the acquisition of problem-solving knowledge in domains like mathematics and physics the added value of abstract diagrammatic and graphical representations has been acknowledged (e.g., Shah & Hoeffner, 2002). These types of visualizations are said to be computationally effective in that they facilitate specific inferential processes needed for some learning tasks (Larkin & Simon, 1987). However, there is no research to our knowing on whether concrete pictures that depict a problem statement and its associated solution procedure promote the acquisition of problem-solving knowledge. Representing the problem statement in a picture might help to understand which object features and interrelations are relevant to the solution of the problem. For instance, in the sprinter example seven sprinters on the racetrack are depicted out of which three can win the gold, silver, and the bronze medal – represented by the pedestal. Additionally visualizing the solution steps may support learners in inferring which change is achieved by applying a solution procedure when they compare the new problem state to the previous one. For instance, comparing the picture illustrating the second solution step to the one of the first step helps to clarify the fact that only six sprinters are eligible for guessing the winner of the silver medal, because one sprinter has already been assigned to the gold medal in the first solution step and thus already stands on the pedestal.

Animations. An animation is „any application which generates a series of frames, so that each frame appears as an alteration of the previous one, and where the sequence of frames is determined either by the designer or the user" (Betrancourt & Tversky, 2000, p. 313). Thus, animations do not only depict the current status of objects; rather they additionally deliver information concerning changes of objects and of their position over time (motion) as well as information concerning the direction of these changes (trajectory, Rieber, 1990). Several findings suggest that animations can be used successfully for delivering abstract contents like mathematical rules, Newton's laws, or computer algorithms (Baek & Layne, 1988; Byrne, Catrambone & Stasko, 1999; Catrambone & Seay, 2002; Rieber, 1990). With respect to conveying problem-solving knowledge the visual-spatial properties of the visualization may be used to deliver information on the current problem state and its relevant structural features. Moreover, the changes over time that can be depicted in an animation may be used to reflect the changes in problem states that result from applying a solution step to a specific problem state of the example – without the need to compare multiple representations as it is necessary when learning from static pictures.

However, learning from animations is known to impose certain requirements onto the learners that they may have difficulty to meet (Betrancourt & Tversky, 2000). Pane, Corbett, and John (1996) have demonstrated that learners often fail to use animations to a sufficient extent and thus miss important information. The results of Lowe (1999) additionally suggest that learners have trouble focusing on the most relevant parts of an animation and are often distracted by salient, but irrelevant details. Furthermore, due to the dynamic changes of the display the information that has to be remembered can only be viewed for a limited amount of time and may therefore have vanished before learners have identified it. Finally, dynamic visualizations may lead to an overly passive information processing and prevent learners from performing effortful cognitive processes required for a deeper understanding (Palmiter & Elkerton, 1993). In line with Salomon's principle of least effort (Salomon, 1984) learners may refrain from deeply processing the contents of an animation and passively watch it as an ongoing movie.

Imagery. To circumvent the problem of passive information processing we implemented a third visualization condition in our experiment in which we instructed learners to imagine the contents of a text-based worked-out example. That is, learners did not receive any pictorial representations at all; rather, they were told to construct their own visualizations. Hodes (1992) compared the effectiveness of imagery instructions and instructional visuals for fact recall and understanding. Both instructional methods were helpful in inducing an imagery strategy and in improving posttest performance; however, for some performance measures achievements due to presenting external visuals were larger than for the imagery instructions. Ginns, Chandler, and Sweller (2003) showed that imagining (vs. studying text-based materials) was helpful only for learners who possessed sufficiently high prior knowledge in the domain. The authors explained this finding by assuming that "to successfully imagine a procedure or a concept, all of the relevant elements must be processed simultaneously in working memory. That may be possible only after a schema had been constructed" (p. 231).

Comparing the three different visualization methods (static pictures, animations, and imagery) to each other yields insights on specific promises as well as on drawbacks associated with these methods. The trade-off between these promises and drawbacks may determine learning outcomes (Table 1). Learning from *static pictures* may engage learners in a more active way, because they have to compare multiple visualizations in order to understand the to-be-learned solution procedures. As this information is permanently visible these comparisons may be conducted without overloading the cognitive system. However, learners may make wrong inferences and they may miss important information so that their internal representation of the solution procedures may be incorrect as well as incomplete. Moreover, because the external representations cannot be modified by a learner they are not adapted to his or her preferences or prior knowledge level. *Dynamic pictures* may on the other hand reduce learner activity and induce a rather passive style of processing. The cognitive demands imposed by the need to extract the relevant information from a changing display and to memorize this information may be rather high. Unless animations are highly interactive they do not allow for any adaptation to a learner's preferences or prior knowledge. However, an advantage of dynamic visualizations of solution steps is that all the information that is needed to understand problem states and their changes is in principle contained in the representation and thus it is correct as well as complete. Finally, *imagery instructions* should foster learner activity in an optimal way; however, the need to envision all information may at the same time require rather demanding processes (Ginns et al., 2003). There is also the danger that learners may miss important information or that they make incorrect inferences. A possible advantage of mental imagery is that because it is based on self-generated images, these images are adapted to a learner's preferences and prior knowledge level.

Table 1: Promises (+) and drawbacks (-) of visualization methods

	Static pictures	Dynamic pictures	Imagery
Learner activity	+	-	+
Cognitive load	+	-	-
Correctness / completeness	-	+	-
Adaptation to learner preferences / knowledge	-	-	+

These trade-offs between possible promises and drawbacks were investigated in the experiment outlined in the remainder of the paper.

2. EXPERIMENT

Method

Participants. Participants were 124 students (88 female, 36 male) of the University of Tuebingen, Germany, who participated for either course credit or payment. Average age was 23.7 years. Most of the subjects were familiar with the domain of probability theory and were experienced computer users.

Materials and Procedure. HYPERCOMB consisted of a technical instruction, a short introduction to the domain, an example-based learning phase and a subsequent test phase. Before starting with the experiment participants filled in a multiple-choice questionnaire with eleven questions on important concepts and definitions from the field of probability theory. This questionnaire was used to measure participants' prior knowledge. In the first part of HYPERCOMB learners were given a short technical introduction to the system and to the experiment. Consequently, the basic notion of random experiments and the general rationale behind calculating the probability of outcomes were explained in a short introduction to the domain. In the subsequent example-based learning phase learners had to acquire knowledge on four different problem categories, whereby each category was explained by means of two worked-out examples. Depending on the experimental condition subjects additionally received an imagery instruction or they could retrieve static pictures or dynamic visualizations that augmented the worked-out examples. Subjects were not forced to process these visualizations; rather, they had to select them by clicking on a button in order to view them. The visualizations depicted the contents of the worked-out examples in a concrete way (cf. Figure 1). For every worked-out example there was always one visualization of the problem statement and one of every worked-out solution step. Following the coherence principle (Mayer, 2001) the visualizations were kept as simple as possible and were not cluttered with any irrelevant details. Participants could decide by themselves when to start working on the test problems. The instructional materials were no longer available during problem solving. For the eleven test problems we varied the transfer distance with respect to the worked-out examples by presenting isomorphic as well as novel problems. Isomorphic test problems differed from the instructional examples only with regard to their surface features. Novel test problems were constructed in a way that two complex-event probabilities had to be considered whose outcomes had to be multiplied in order to calculate the required probability.

Design and Dependent Measures. Subjects learned in one of four instructional conditions. In the text-only condition only the written worked-out examples were available. In the imagery condition learners were additionally told to mentally imagine the contents as vividly and with as many details as possible. Mental imagery was trained at the beginning of the experiment by instructing learners to imagine the contents of a short text passage describing a traffic situation involving multiple cars which approach a crossing from different directions. Moreover, they were constantly reminded to use imagery in the example-based learning phase by a sticker "imagine the situation" attached to the computer screen. In the pictures condition static visualizations could be retrieved for each component of the worked-out examples, whereas in the animation condition clicking the play-button for any of the example components resulted in the presentation of a dynamic visualization. For instance, in the sprinter animation the sprinters were first entering the racetrack (problem statement) and were then running across the racetrack. The visualization of the solution steps always depicted one sprinter passing the finishing line and ascending the pedestal. As static pictures we always used the last frame of the animations (cf. Figure 1) that depicted the problem state resulting from the application of the respective solution step.

As dependent measures we registered problem-solving performance for isomorphic and novel test problems and the time spent on learning (learning time).

Results

Overall Analyses. Prior knowledge was comparable across all four instructional conditions. In a first step we analyzed problem-solving performance and learning time across all four conditions by means of one-factorial ANOVAs. Performance on isomorphic problems (Figure 2, left) varied slightly as a function of instructional condition ($F(3,120) = 2.19$; $Mse = 783.90$; $p < .10$), while performance on novel problems (Figure 2, right) was left unaffected by the experimental manipulation ($F < 1$). Learning time (Figure 3) increased rather naturally with the more instructional materials being available for processing ($F(3,120) = 5.36$; $Mse = 55915.01$; $p < .01$).

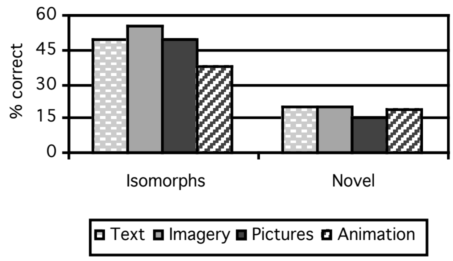

Figure 2: Problem-solving performance (in % correct) for isomorphic (left) and novel problems (right) as a function of instructional condition

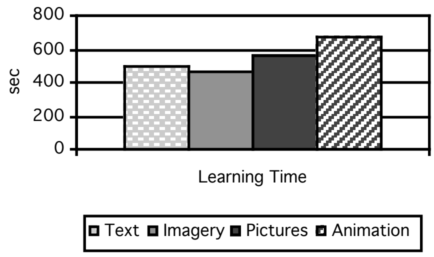

Figure 3: Learning time (in sec) as a function of instructional condition

Thus, at first sight it seems that although presenting external visualizations increased the time learners devoted for learning, these increases in learning time were not accompanied by respective gains in performance. On the contrary, performance for isomorphic problems was worst in the animation condition. However, it is not clear whether the finding that problem-solving performance was only slightly affected by variations of the instructional materials is due to the ineffectiveness of these variations or whether it is due to the fact that learners did not sufficiently use the external visualizations. To address this question we analyzed the frequency by which learners retrieved these visualizations in a next step.

Utilization of External Visualizations. For every subject we determined how often he or she had retrieved visualizations by clicking on the respective button in the two visualization conditions. There were a total of 39 visualizations available in each of the conditions; however, these were only seldom retrieved (Figure 4). Static pictures were retrieved 6.6 times on average, whereas animations were played 10.9 times. This difference between the two conditions was marginally significant in a two-tailed t-test ($t(57) = 1.71$; $p < .10$). For the further course of the statistical analyses we conducted a median split within each of the two conditions to distinguish between learners who used external visualizations only sparsely and those who made frequent use of the representations. The resulting variable was used as a second factor in an ANOVA (instructional condition x visualization utilization).

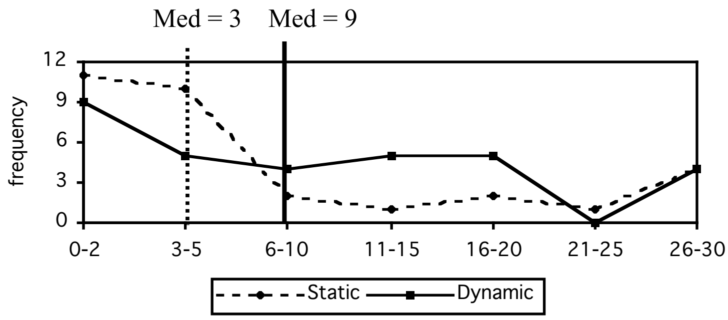

Figure 4: Frequency of using external visualizations in the static and in the dynamic visualization condition

Analyzing performance in the conditions with external visualizations by means of this ANOVA revealed no significant effects for performance on novel problems (all Fs < 1). However, for performance on isomorphic test problems (Figure 5) there were marginally significant main effects of external visualization and visualization utilization. Learners showed a slightly superior performance when learning from static pictures ($F(1,55) = 3.66$; $Mse = 753.00$; $p < .10$). Furthermore, learners who made use of the visualizations more frequently outperformed those who only used them sparsely ($F(1,55) = 3.59$; $Mse = 753.00$; $p < .10$). Finally, there was a significant interaction ($F(1,55) = 5.12$; $Mse = 753.00$; $p < .05$) indicating that – when frequently used – static pictures were superior to animations ($t(25) = 2.64$; $p < .05$), while there were no differences when the visualizations were retrieved only sparsely ($t(30) = -0.28$; $p > .70$).

Figure 5 additionally displays the performance in the text-only condition as a baseline. Contrasting the various conditions with this baseline indicated that only the frequent use of static pictures resulted in performance improvements that were significant at the .05 level. All the other conditions achieved a performance that lay below the baseline performance that learners were able to achieve when studying the text-based worked-out examples only.

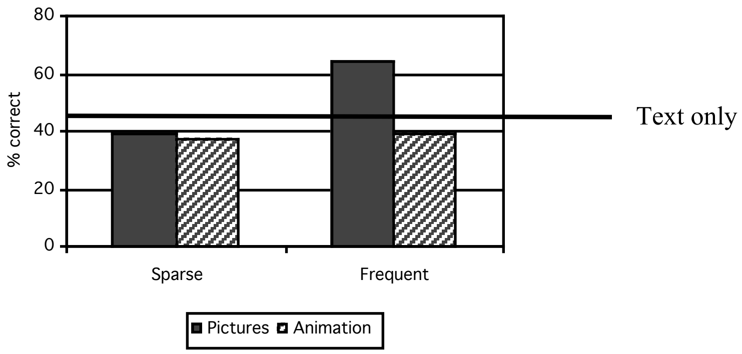

Figure 5: Performance on isomorphic problems (in % correct) as a function of external visualization and visualization utilization

Furthermore, as an analysis of the learning time (Figure 6) revealed this deterioration in performance due to frequently using animations was accompanied by an increase in time learners needed to study the instructional materials. The type of external visualization had no impact on learning time ($F < 1$), whereas learning time certainly increased with a more intensive use of external visualizations ($F (1,55) = 15.77$; $Mse = 62160.57$; $p < .001$). As indicated by a marginally significant interaction these increases in learning time due to a frequent retrieval of external visualizations were especially apparent in the animation condition ($F (1,55) = 2.89$; $Mse = 62160.57$; $p < .10$). That is, in the pictures condition learners who frequently used visualizations did not differ from those using them only sparsely with regard to learning time ($t(27) = 1.58$; $p > .10$), whereas using animations frequently lead to highly significant increases in learning time ($t(28) = 4.26$; $p < .001$).

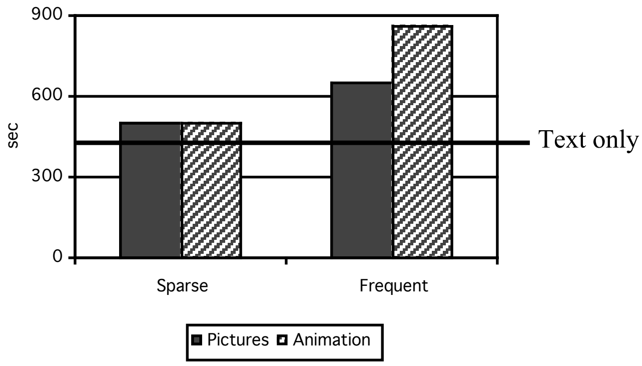

Figure 6: Learning time (in sec) as a function of external visualization and visualization utilization

To sum up, using static pictures frequently did not increase learning time demands and nevertheless improved performance. On the contrary, frequently using animations was rather damaging in that it required more learning time and did not yield any performance improvements. Therefore, while concrete visualizations of the contents of the worked-out examples proved beneficial for learning, the dynamics contained in the animations were unnecessary or even harmful. If complex dynamic visualizations cannot be recommended for learning, the question arises whether there is a need for external visualizations at all. That is, if already simple visualizations like static pictures of the content help to achieve an understanding of the principles, then maybe learners are able to generate these images by themselves. The question whether external visualizations are more helpful than the instruction to imagine the contents of the worked-out examples was addressed in the last analysis.

The Benefits of Imagery Compared to External Visualizations. In this final section we compared the imagery condition to the two external-visualization conditions whereby in the latter two we further distinguished between sparse and frequent use of visualizations. Because none of the analyses revealed any effects for performance on novel problems we will report the results for isomorphic problems only.

There were no differences with regard to performance on isomorphic problems between the imagery condition and the two subgroups who learned from static pictures (imagery vs. sparse picture use: $t(45) = 1.42$; $p > .15$; imagery vs. frequent use of pictures: $t(41) = -1.27$; $p > .20$). Using animations frequently was worse than receiving an imagery instruction only ($t(46) = 1.97$; $p < .10$), while there were smaller differences between the sparsely use of animations and the latter condition ($t(44) = 1.64$; $p > .10$). With regard to learning time, the frequent use of external visualizations increased the time spent studying the instructional materials compared to the imagery instruction (imagery vs. frequent picture use: ($t(41) = -2.69$; $p = .01$; imagery vs. frequent animation use: $t(46) = 6.82$; $p < .001$), whereas the sparse use did not yield any longer learning times (imagery vs. sparse picture use: $t(45) = 0.15$; $p < .80$; imagery vs. sparse animation use: $t(44) = 0.52$; $p > .60$). Thus, imagining the contents of the worked-out examples was as effective as viewing static pictures that depicted these contents and more efficient because it required less time for learning. Moreover, mental imagery was slightly more effective than receiving dynamic visualizations and was again accompanied by less learning time.

3. CONCLUSIONS

In this paper we presented evidence for the differential effectiveness of external and internal visualizations for cognitive skill acquisition. Our results support the assumption that learners might benefit from a concrete visualization of problem states that is tied to the cover stories of worked-out examples. However, while we were able to show that a frequent use of static pictures fostered performance at least on isomorphic problems, including dynamics in the external visualizations worsened performance. The initial idea of using animations had been that the dynamics of an animation might be used to depict information on changes in problem states that occur due to applying a specific solution step. However, it seems that representing these changes explicitly was more harming than helpful in that it may even have distracted learners. These results are in line with prior findings (Lowe, 1999; Pane et al., 1996) showing that learners are often overwhelmed by the number of details they need to identify, select, and to remember in a limited period of time (i.e., while the information is present on the changing display) when learning from animations.

In our current research we thus pursue the idea of further reducing the cognitive demands imposed by animations by using abstract rather than concrete dynamic visualizations of the worked-out examples. These abstract animations are characterized by the fact that the visualizations of all examples share a common representation of objects and of the relevant relations among them. That is, irrespective of an example's cover story objects like the sprinters are represented by marbles which are selected from an urn. This common representation across examples should help learners to focus on the structural similarities and differences between the examples while being able to ignore their surface features. Thus, there is not only less information that needs to be processed in total; moreover, the ratio between relevant and irrelevant information is improved compared to the concrete animations investigated in the current paper. Thus, this simplified representation should be less demanding and should leave free cognitive resources in order to cope with the dynamics of the animation.

Additionally, we want to continue our work based on the promising results with regard to cognitive skill acquisition from static pictures and imagery. First, we would like to investigate means of improving learners' use of static pictures. In particular, we aim at testing the effectiveness of *retrieval prompts* which guide learners to use static pictures more often. In prior studies we were able to demonstrate that prompting learners to retrieve profitable information units is an effective means to foster problem-solving performance in particular for students with low prior knowledge (Gerjets, Scheiter, & Schuh, 2004). Second, once static pictures have been retrieved to a sufficient extent, their processing should be supported by additional instructional guidance. In particular, learners may receive *instructions to compare multiple static pictures* to enable them to extract changes in problem states that have occurred due to applying a solution step more easily. Finally, we are also convinced that the use of imagery might be further improved. In the current experiment learners were instructed to use imagery only at the beginning of the experiment and the reminder during the learning phase could be easily overlooked. Thus, we would like to investigate the use of computer-based prompts that frequently remind learners to envision the examples' contents. All three instructional devices (retrieval prompts, comparison instruction, and imagery prompts) aim at increasing the time learners devote for studying the instructional materials as well as at improving the quality of processing the worked-out examples.

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