An Interactive, Multimedia Environment for Exploring Tonal Pitch Space

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Abstract:

Because they depict relations among musical pitches, chords, and key, charts in Fred Lerdahl's Tonal Pitch Space *can be rendered more accurately and vividly with multimedia authoring software. My renderings enable a user to orient charts toward any specific key, trace any of numerous possible paths, determine the depth of embedding of any element in any tonal context, and compute distances between elements. Lerdahl's analyses can be depicted more vividly by synchronizing graphic images with the corresponding music. Graphic representations have proven to be effective instructional tools, especially when used to illustrate abstract concepts. Multimedia realizations should prove even more effective when used in appropriate contexts.*

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1 Introduction

Within the discipline of music theory, there is a long history of using spatial analogies to explain relations among pitches, pitch combinations, and keys[i]. Fred Lerdahl's *Tonal Pitch Space*[ii] is a recent and major contribution to this tradition. Building upon a theoretical foundation laid in a previous treatise[iii], Lerdahl has refined and extended his account of how listeners comprehend music composed in the system of pitch organization known to music theorists as functional tonality. In doing so, he has drawn extensively upon recent research in music cognition and perception, especially the work of Carol Krumhansl and her associates.[iv] By engaging experimental studies that corroborate his theories and revising his theories to account for recent findings, Lerdahl has demonstrated a concern for psychological validity that is atypical of much recent theorizing about music.

Since Lerdahl present his theories in traditional book format, his figures are necessarily restricted to two-dimensional space. Some require only two dimensions, but others need three. Many of his models can be rendered more accurately and vividly with multimedia authoring software. In this paper I will discuss and illustrate the results of my efforts in this regard.

2 Interactive Models

2.1 Abstract models of spaces and paths

Throughout his book, Lerdahl provides charts to illustrate how tonal entities, such as pitches, pitch classes (pcs), chords, and keys, relate to each other within their respective spaces. Some of his charts are oriented toward one specific key with the implication that they could be transposed to any of the twenty-four major and minor keys. The cognitive demands of that task would be light for professional music theorists, but they would be much heavier for novices. My realizations lighten this burden by providing buttons that enable a user to orient a chart toward any key or tonal context. Once the desired chart is displayed, the user can trace any possible path through its space. Figure 1 shows an interactive animation that enables one to determine the depth of embedding and voice-leading function for any pitch class (pc) in any major or minor key. Using this model, an instructor could show, or a student could discover, that pitch classes (pcs) with the same letter-name function differently in various keys, and conversely, that pcs with different letter-names can function identically within various keys. Figure 2 shows one state of an interactive movie that enables one to apply Lerdahl's algorithm for computing the distance between any two chords within any region (key).

Figure 1. Lerdahl's basic space (*Tonal Pitch Space*, Fig. 2.5c, p. 49).

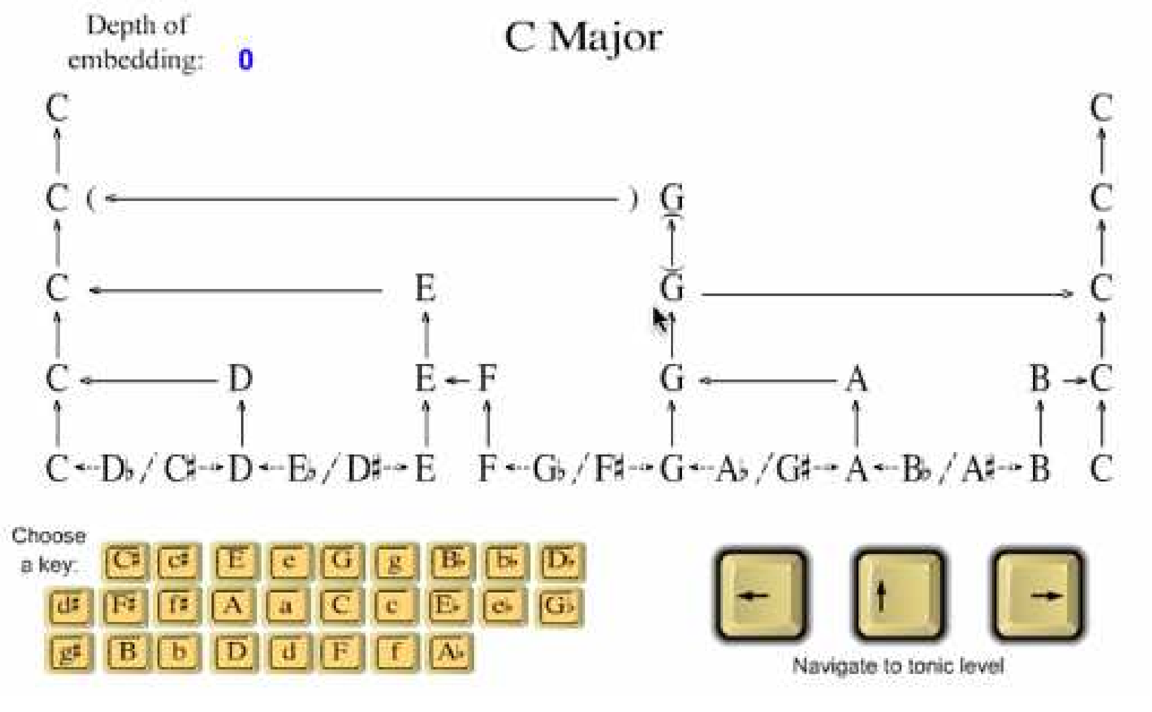
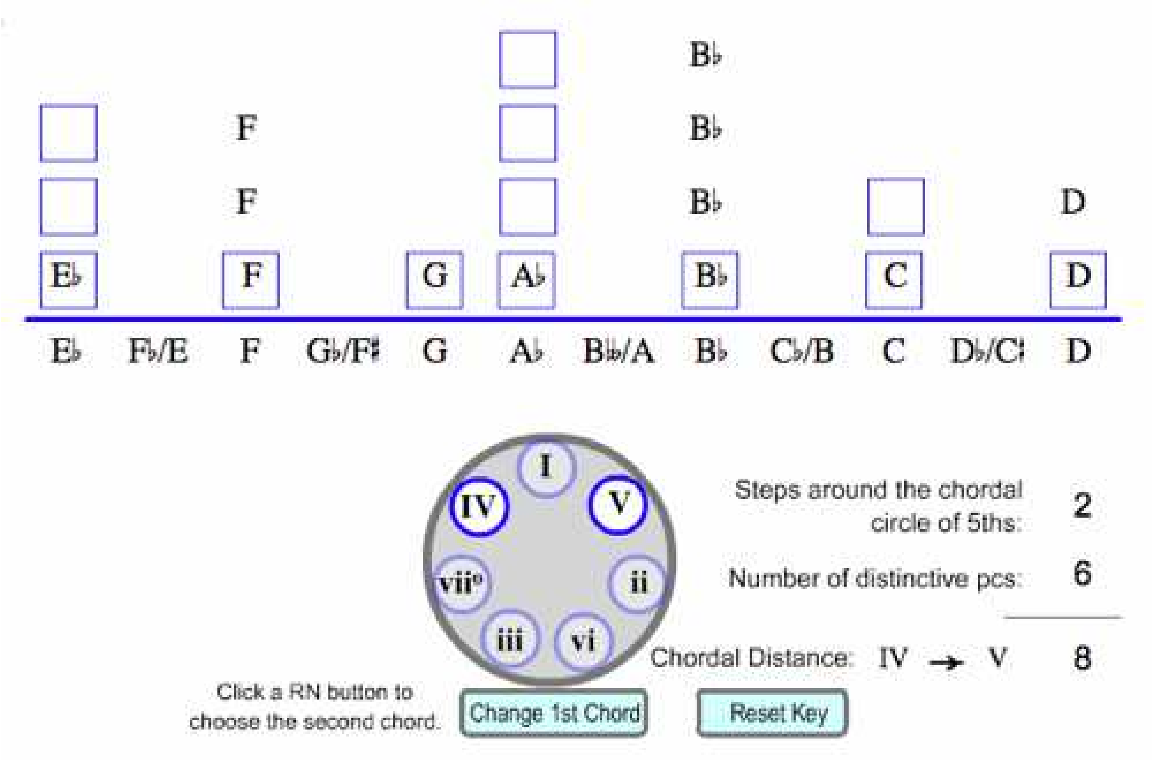


Figure 2. Computing chordal distances within a key (*Tonal Pitch Space*, Fig. 2.10, p. 56)



Lerdahl conceives chordal space as cyclic in both the horizontal and vertical dimensions, but he nevertheless represents it on a flat surface.[v] The cyclic property of chordal space can be depicted more vividly by plotting Roman numerals, which represent the position of a chord's root on the scale of the prevailing key, on two cylinders. These figures can be rotated on their respective horizontal or vertical axis (see Fig. 3). To obtain a more accurate model the two cylinders can then be joined to form a torus, which can be rotated on both axes (Fig. 4).

Figure 3. Chordal space plotted on two orthogonal cylinders.

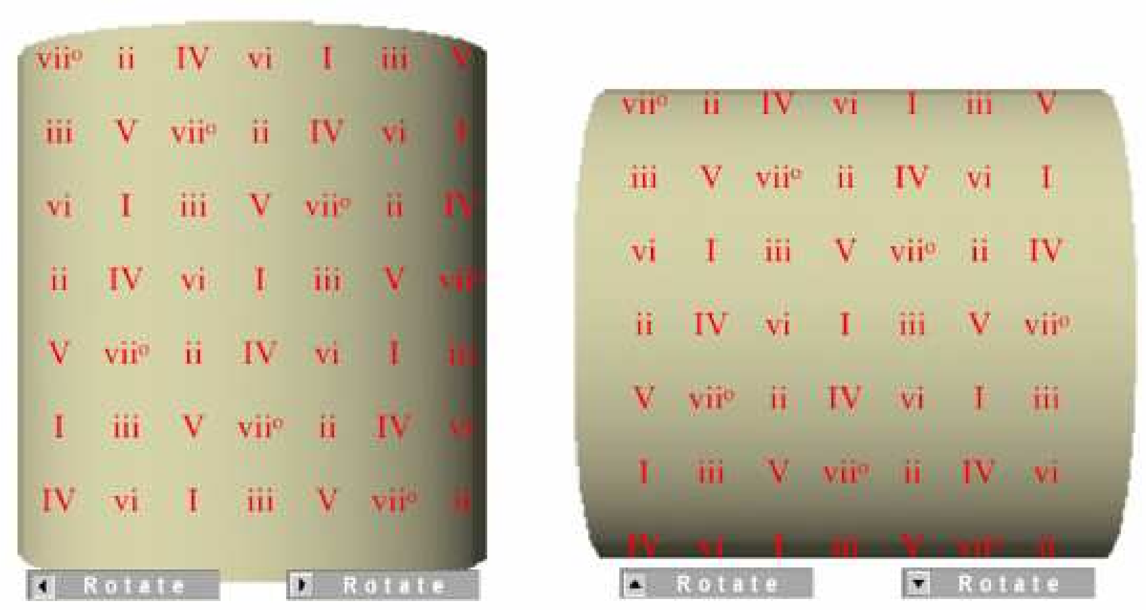


Figure 4. Chordal space plotted on a torus

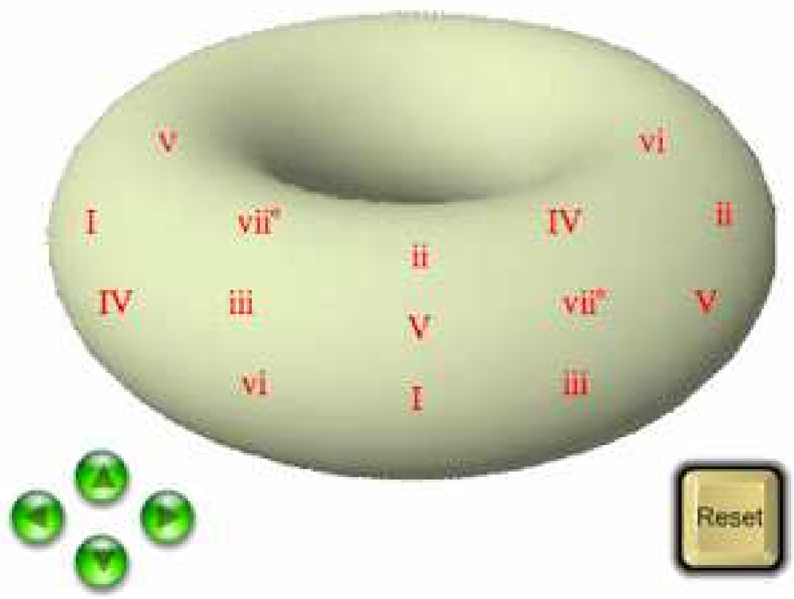
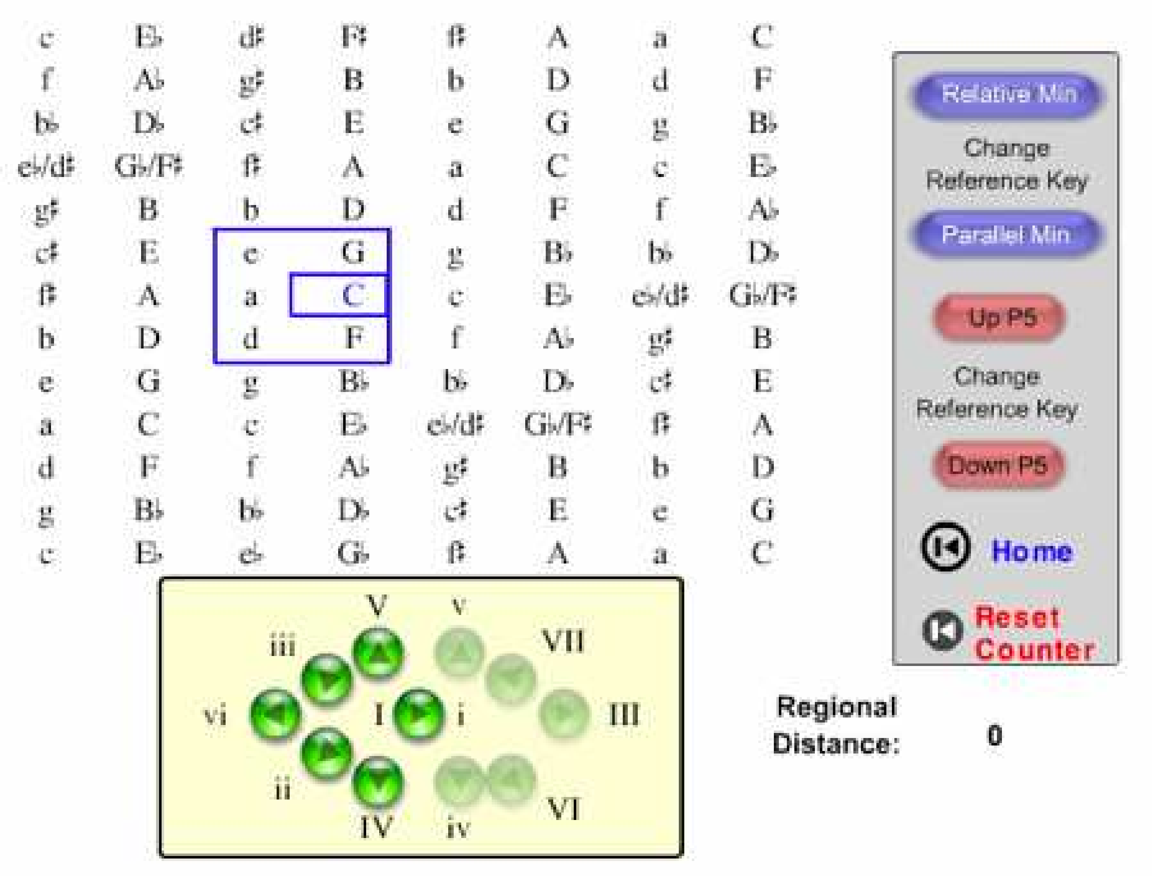


Figure 5 shows a movie for exploring various paths through a flattened version of Lerdahl's chordal and regional space. After setting the key of origin, the user can click arrow buttons to navigate vertically along the axis of fifth-related keys, or horizontally along the axis of third-related, relative and parallel keys, from the key of origin to any other key. In doing so, he/she discovers that the number of possible paths between two keys increases with the distance between those keys.

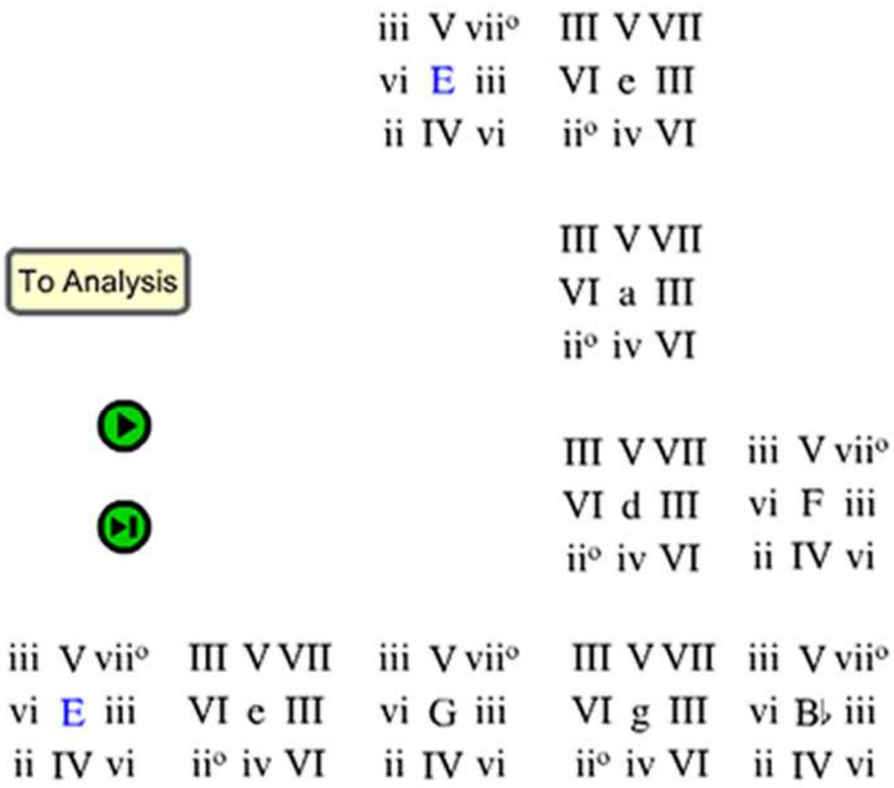
Figure 5. Paths and chord distances within regional space (*Tonal Pitch Space*, Figs. 2.25 and 2.26)



2.2 Interactive analyses

To demonstrate the explanatory power of his theories, Lerdahl applies them in analyses of specific works or passages. Several of his analyses consist of figures, charts, and/or diagrams of the relevant pitch space along with descriptions of various paths traversed through that space. Since music must be experienced over spans of time, these analyses can be rendered more vividly by synchronizing graphic images with the musical events they represent. In addition to engaging both the visual and aural senses, such realizations enable a user to step through an analysis so that he/she may pause to ponder the relation between any pair of successive chords. Figure 6 shows Lerdahl's depiction of the path traversed through chordal/regional space during the third phrase of Chopin's Prelude in E Major. Paths traversed during the first and second phrases of this prelude are depicted with similar "animated analyses."

Figure 6. Chordal-regional space traversed during phrase 3 of Chopin's Prelude in E Major, Op. 28 (*Tonal Pitch Space*, Fig. 3.8, p. 98).



2.3 Tonal tension and attraction

Lerdahl addresses the issues of tonal tension and attraction in Chapter Four of *Tonal Pitch Space*. He begins by using time-span and prolongational reductions to depict the sequential and hierarchical distance between chords in bars 1-9 of the first movement of Mozart's Piano Sonata in Eb Major, K. 282. His figures provide the relevant score notation and tree diagrams, but they do not show how the distance for each chord is computed. (Those values are displayed in tabular format elsewhere.) My realizations allow the user to step through the passage and view the computation of each distance value, or play the passage and observe the values being computed as the music unfolds (see Fig. 7). Using another movie, the user can compute the hierarchical tension of any event while viewing a tree diagram that shows how that a relatively unstable event inherits tension from more stable, superordinate events (see Fig. 8). One can also plot the line graph for sequential tension, or hierarchical tension, or for both values. These graphs can be plotted one chord at a time or in real time as the music plays.

Figure 7. Time-span reduction with sequential distance calculations (*Tonal Pitch Space*, Figs. 4.2, 4.3, and 4.4)

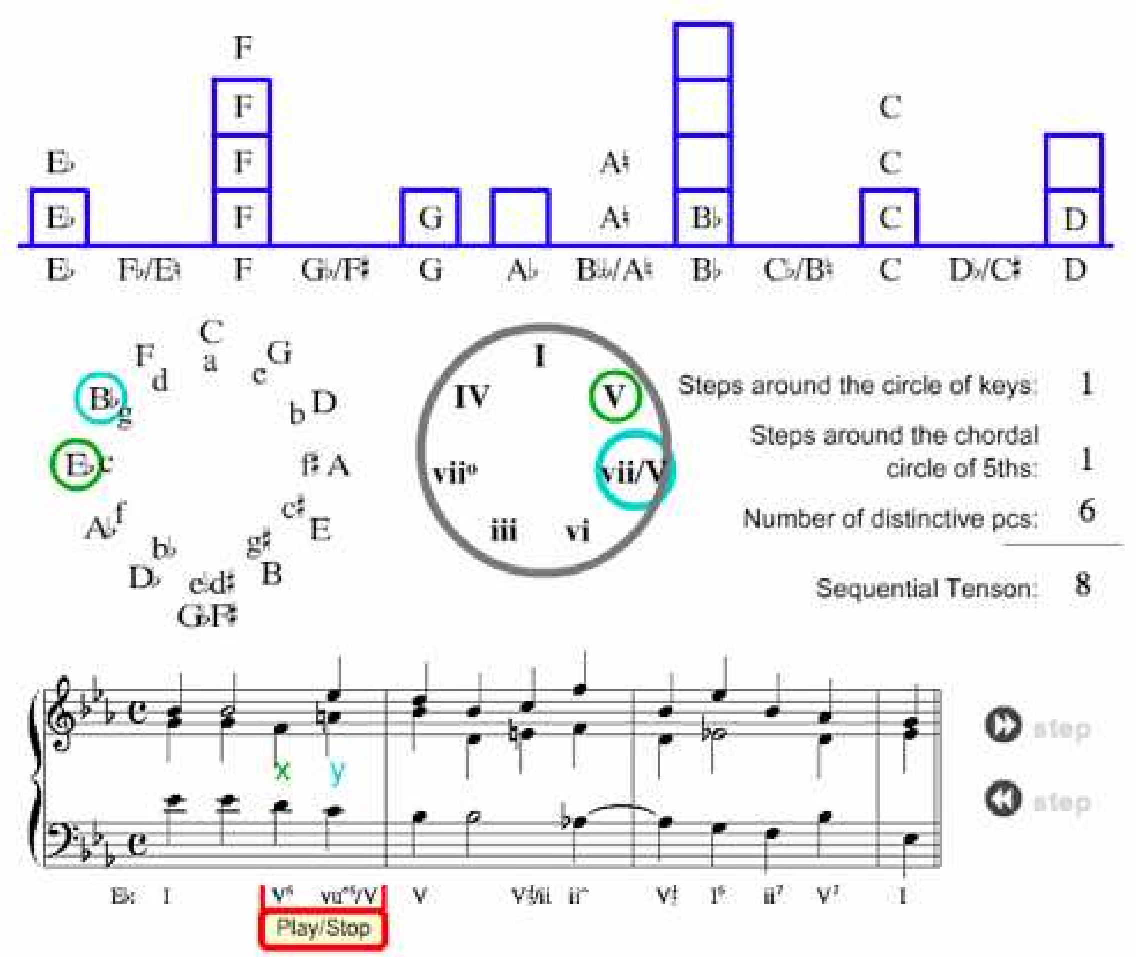
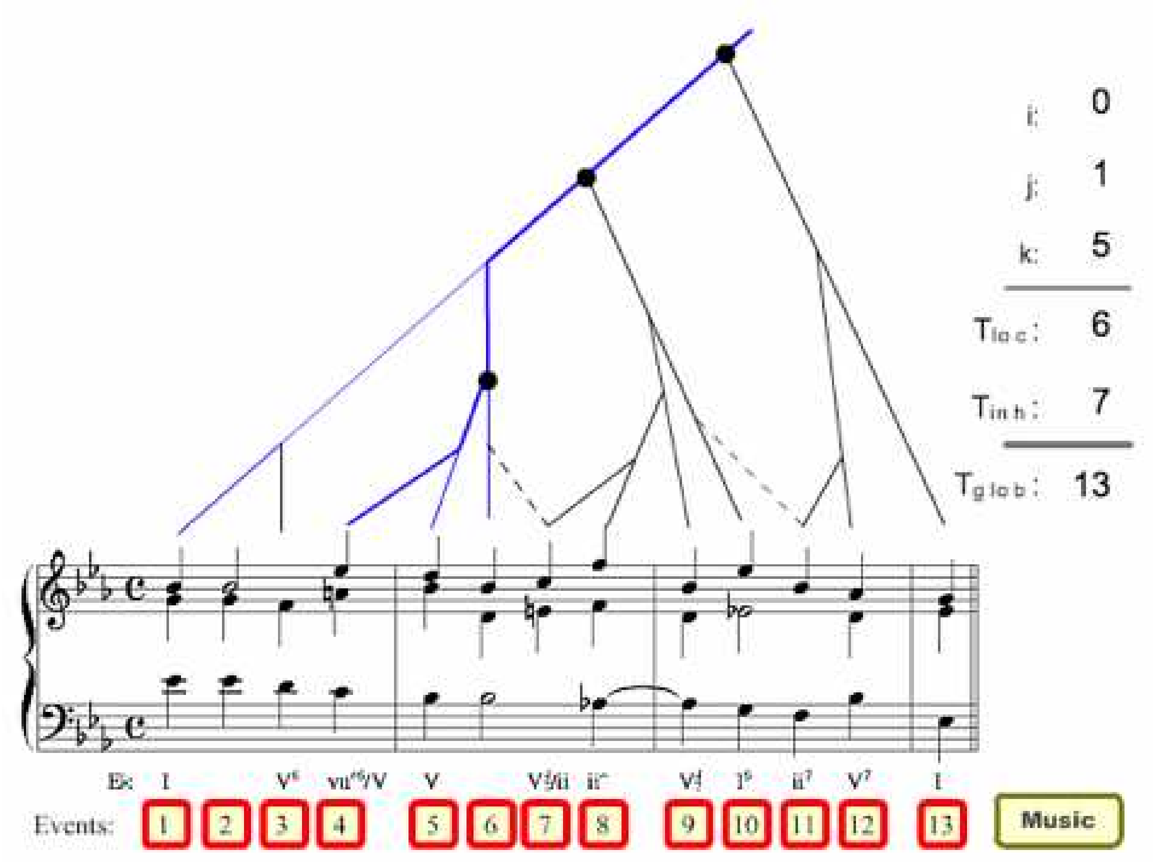
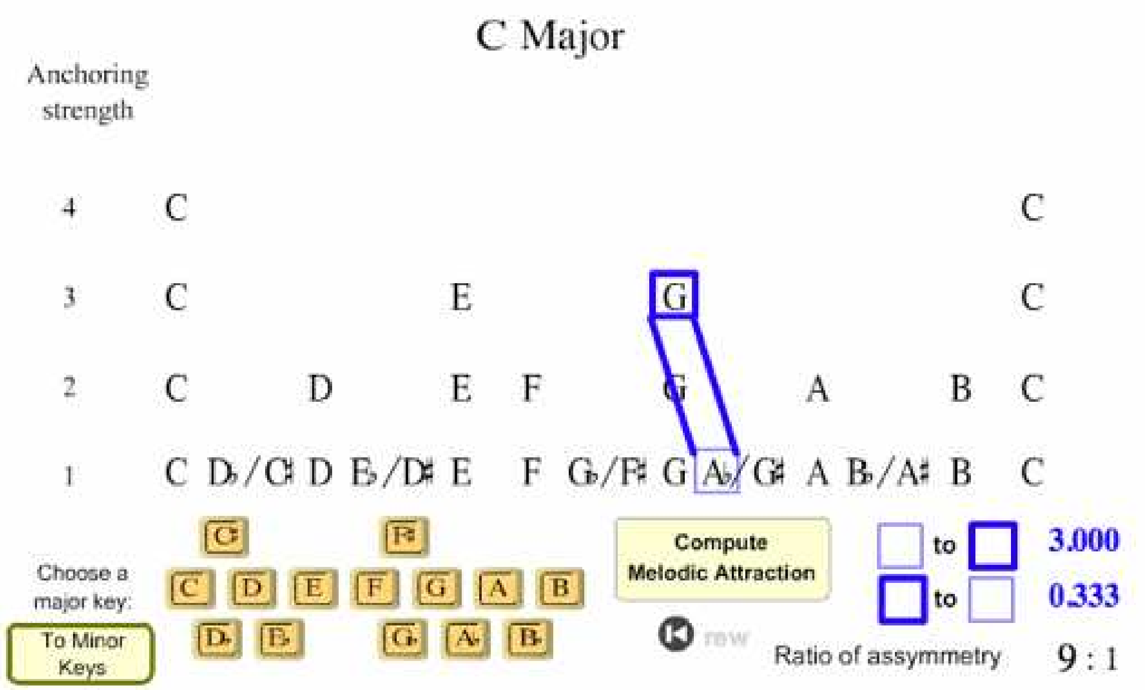


Figure 8. Prolongational reduction with hierarchical tension calculation (*Tonal Pitch Space*, Fig. 4.5)



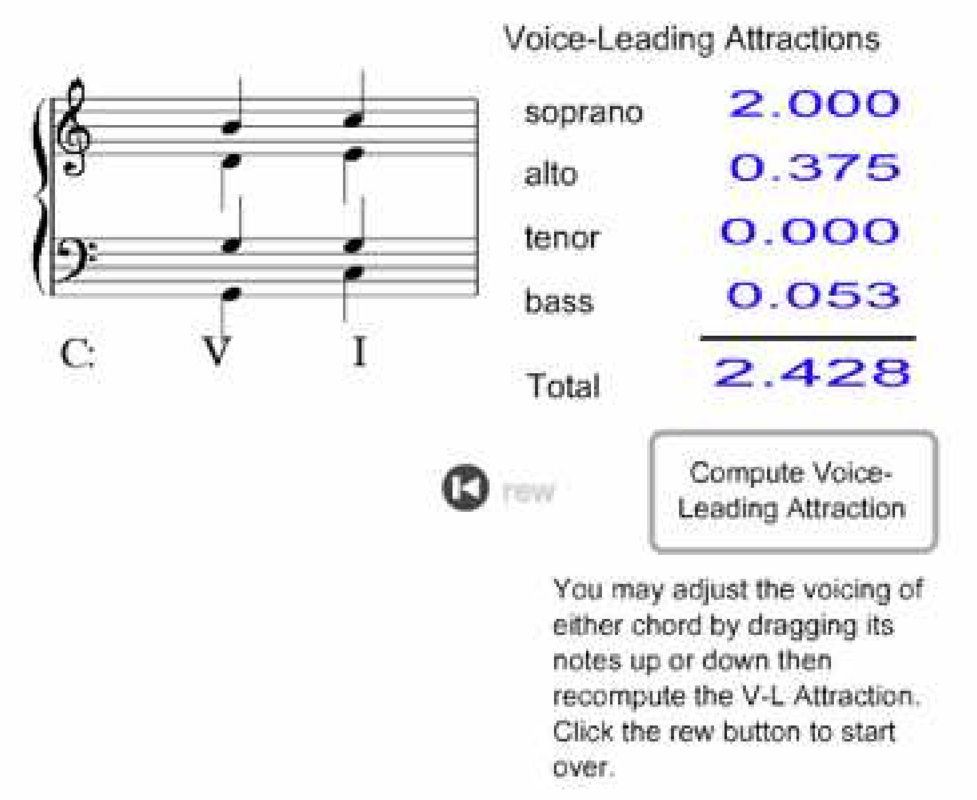
Lerdahl's algorithms for computing melodic and voice-leading attraction can also be depicted with interactive animations. Figure 9 shows a movie for computing the melodic attraction between any two pitches located on different levels of the basic space. Because two such pitches have different anchoring strength, their mutual attraction is asymmetrical. The attraction of the more stable pitch to the less stable pitch is the reciprocal of the anchoring strength ratio of the less stable pitch to the more stable pitch. To account for the effect of distance, each ratio is multiplied by the inverse square of the distance in semitones (n) between the two pitches, p_1 and p_2. The complete formula for melodic attraction is, therefore: $\alpha(p_1 \rightarrow p_2) = s_1/s_2 \times 1/n^{2,}$ where s_1 is the anchoring strength of p_1 and s_2 is the anchoring strength of p_2.[vi] As Fig. 8 shows, the animation computes the attraction values in both directions and displays the ratio of asymmetry.

Figure 9. Attraction between pitch classes with different anchoring strength (*Tonal Pitch Space*, Fig. 4.25, p. 166).



According to Lerdahl, the voice-leading attraction between consecutive chords is the sum of the melodic attraction between the tones of the first chord and the corresponding tones of the second chord. Figure 10 shows an animation for computing the voice-leading attraction between any two diatonic triads in the key of C major. When the user clicks on Roman numeral buttons, the tones of the corresponding chords are plotted on the grand staff in a standard voicing. The voice-leading attraction of the first chord to the second can be computed by clicking another button. The user may then alter the voicing of either chord by dragging one or more of its notes up or down on the staff and then re-compute the attraction values. In doing so, he/she can develop an appreciation for how smooth voice leading enhances the attraction between two chords. A similar movie is provided for computing voice-leading attractions in minor-key contexts. The overall realized harmonic attraction of one chord for another is computed by summing the voice-leading attractions of the various voices and dividing that sum by the distance between the two chords as measured in Figure 2 above. Another set of movies is provided to illustrate this two-stage computational process.

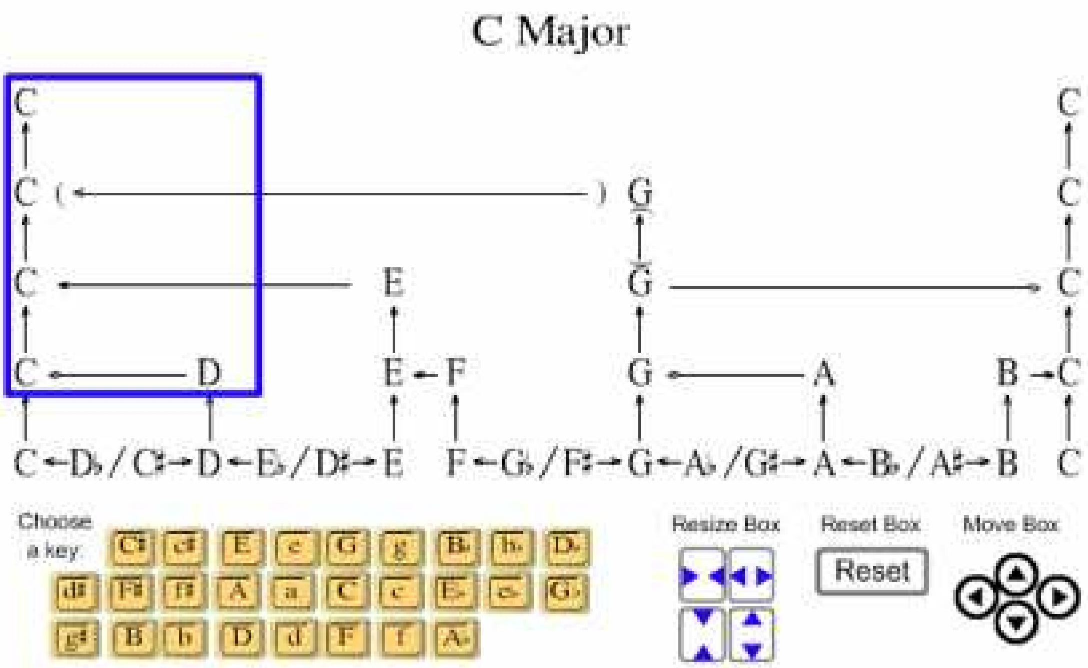
Figure 10. Voice-leading attraction between two diatonic triads in C major (*Tonal Pitch Space*, Fig. 4.29a, p. 172).



2.4 Finding the tonic

At the beginning of Chapter Five, Lerdahl presents his theory of how listeners gain their tonal bearings and eventually locate the tonic scale degree of a musical passage. Briefly, he claims that when someone hears a sequence of two pitches, he/she evaluates the relative stability of those pitches in various possible tonal contexts. In spatial terms, that task can be represented by situating two pitches within the space of the key in which they have the greatest stability. When two diatonic pitches are plotted in the basic space, one's sense of key orientation is determined by each pitch's depth of embedding. Lerdahl shows how tonic orientation might be determined for eight different sequences of two pitches.[vii] My realization (see Fig. 11) enables one to apply Lerdahl's tonic-finding algorithm to any sequence of two diatonic pitches in any of the twenty-four major and minor keys.

Figure 11. Tonic orientation for sequences of two pitches (*Tonal Pitch Space*, Fig. 5.1, p. 195)



3 Pedagogical Effectiveness

The interactive "movies" described above were developed in close consultation with Prof. Lerdahl. Although their pedagogical effectiveness has not been tested, my experience to date leads me to predict that they can enhance comprehension of his theories, especially among novices.

In a recent summary of the effect of instructional variables upon learning Wallace Hannum[viii] noted that graphic representations typically produce an effect size of about 1.24 standard deviation units above the norm (see Table 1), and that the effect is even greater when graphic models are used to illustrate abstract concepts. It seems reasonable to assume, therefore, that this effect would be amplified when such models are used in learning environments that encourage interactive experimentation and engage the learner's senses of sight and hearing. The models that I have discussed were designed to encourage learning in such environments.

Table 1. Effect sizes for different instructional variables (Hannum 2007, Table 2, p. 9)

Variable	**Effect Size**
1: 1 Tutoring	2.00
Analogies	1.65
Comparison	1.32
Graphic Representations	1.24
Generative Activities	1.14
Note-taking	0.99
Corrective Feedback	0.94
Direct Instruction	0.93
Reciprocal Teaching	0.86
Homework	0.77
Cooperative Learning	0.59
Mastery Learning	0.53
Frequent Testing	0.48
Concept Mapping	0.45
Advance Organizers	0.44

[1] J. Lester: *Compositional Theory in the Eighteenth Century*, Harvard University Press, 1982.
[2] F. Lerdahl: *Tonal pitch space*, Oxford University Press, 2001.
[3] F. Lerdahl and R. Jackendoff: *A generative theory of tonal music*. MIT Press, 1983.
[4] C. Krumhansl: *Cognitive foundations of musical pitch*. New York: Oxford University Press, 1990.
[5] *Tonal pitch space*, Fig. 2.14, p. 57.
[6] Tonal pitch space, p. 163.
[7] *Tonal pitch space*, Fig, 5.1, p. 195.
[8] W. H. Hannum. When computers teach: A review of the instructional effectiveness of computers. *Educational Technology*, March/April, 2007. Table 2, p. 9.

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