Hypermedia as a means for learning and for thinking about learning

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INTRODUCTION

This paper is about the design and realization of an experimentation with a hypermedial system, IPER-3, facing the three 'classical' problems in the history of mathematics (trisection of the angle, quadrature of the circle, duplication of the cube) and some issues which have developed around them. The aim of our work is to study the opportunities provided by hypermedial systems in dealing with topics of interest for the training of teachers and/or for class work, exploiting the possibility of a non-linear presentation, representation of heterogeneous pieces of information, modularization, increment and flexibility of use, as discussed in the literature (Conklin, 1987; Tomek et al., 1991).

The paper is organized as follows. First of all we motivate the choice of the history of mathematics and, inside this, the choice of the three classical problems. Then we present the structure of the hypermedial system IPER-3 and some design and implementation choices which are interesting from a teaching viewpoint. In the last section we analyze an experience of actual use of the system and delineate some possible developments of this type of activity.

MOTIVATIONS

The field of knowledge of the history of mathematics appears particularly suited to representation of a hypermedial type. At the level of both research and, with the due change of scale, work in class, tracing the history of mathematics is a complex activity in which there coexist two different investigation needs, the purely 'informative' one (characters, events, dates, places, etc.) and the 'conceptual' one of the analysis of ideas. Discovering history also means consulting sources including illustrations, manuscripts and various documents, and, since the same topic can be viewed in various milieus and in various epochs, it also means referring to history in general, art, geographical and ethnic situations, etc. In common texts of the history of mathematics, because of the very structure of the book, it is very difficult to handle the non-linearity of the historical development of ideas and hence, for the non-professional historian (for example, a student or a teacher) it may prove difficult to grasp the various connections or the influence of the context and, consequently, also the thread linking the developments of these ideas.

In making our choice to work with a hypermedial system in the history of mathematics we have taken into account, on one hand, these aspects of history and, on the other, the specific characteristic that hypermedial systems have of lending themselves to presentations of a different type than the book, being more dynamic and flexible in both spatial and temporal representation.

The association between the history of mathematics and computer technologies is not a new one. Apart from the massive use made of databases for the organization of historical material, which here does not concern us, it must be remembered that much information of a historical type is available on the Internet. Some students who have participated in the experience have used this resource for getting first information on certain historical facts, which they have then compared with other (more canonical) sources.

Apart from the reasons linked to the specific characteristics of the medium used, the choice of the history of mathematics is also connected to our opinion on the role that this discipline has in the teaching of mathematics (Furinghetti, 1997). We believe that this role is central in the training of the mathematics teacher, in an outlook which has to do not only with knowing facts, which is certainly important, but above all with epistemological reflection as an integral part of the teacher's craft. In this sense history may be one of the contexts which can be activated both for constructing concepts and for attaining awareness of the epistemological obstacles lying behind the difficulties met. These considerations concern not only the training of teachers, but also classroom practice.

Within history the choice fell on the three classical problems for a 'structural' reason: they are a multifaceted subject, a catalyst of many theories and many cultural contexts but one which for this very reason is mortified by the traditional presentation. Moreover, we were interested in the teaching implications of the three classical problems. These implications are linked to their cultural importance which is due to the fact that mathematicians attempted to solve them using the rule and the compass. Only in modern times it was proved that this way of solving them is not possible. The importance of this subject, from the teaching viewpoint, is founded on the belief that it is necessary to account for the reason why algebraic structures were introduced.

In the way of presenting the historical content we bore in mind the exploratory character of this work and we did not choose to consider a single type of user. For this reason, on one hand advanced developments are suggested, while on the other the language and form of presentation are conceived in such a way as also to be accessible to secondary school students, and some space has been devoted to topics of an elementary type (e.g. the detailed explanation of certain geometrical constructions).

THE IPER-3 SYSTEM

As already stated, the IPER-3 system deals with the quadrature of the circle, the trisection of the angle and the duplication of the cube. The present prototype only gives full development of the quadrature of the circle. Work is still in progress in relation to the other two problems.

Information islands and entry routes

IPER-3 consists of a structured set of information islands. Here the term 'information island' is used to mean an organized set of pieces of information which can be accessed by means of hot-words, buttons and sensitive areas. Each information island corresponds to one of the topics dealt with. It is articulated in accordance with various 'entry routes'. An entry route can be specific to one information island or common to several. The information islands are six: 'introduction', 'classical problems' (one for each), 'rule and compass', 'glossary'. It is to be observed that the pieces of information thus organized are interconnected.

The introduction allows one to obtain general information on the system and on the three problems dealt with in it. From this information island one can get to the specific ones for each of the three classical problems. The latter are structured in accordance with four entry routes: 'general information', 'historical pathway', 'solutions', 'solvers'. In turn, the four entry routes are structured in accordance with semantics which will be briefly explained afterwards.

The general information entry route identifies information of a general character (presentation of the problem, formulations of the problem, cultural references, anecdotes, etc.) relating to the problem chosen. The historical development entry route identifies a section presenting some of the main contributions to the study of the specific problem in various periods. The solution entry route is structured in accordance with the type of approach followed for the solution (geometrical, analytical). The solver entry route is closely linked to the previous one: it concerns the instruments used to arrive at the solutions proposed (mechanical, geometrical, etc.).

It was felt to be useful to devote a specific information island to the rule and the compass in order to accentuate the link between the subject we are dealing with and Euclidean geometry.

The information island relating to the glossary is made up of information sheets briefly explaining some important terms. One accesses the glossary by selecting the words or phrases which in the text are boxed (e.g. 'geometrical constructions', 'absolute field of rationality', etc.).

For the mathematical and/or historical contents to put into the system use was made not only of the specific literature but also of notes on teaching experiences in classes at upper secondary schools, and of the Internet. The latter constitutes a work instrument supplying an enormous information patrimony which is continually expanding, though with some limits from the historical viewpoint (frequent lack of bibliography, approximate notices).

The interface

The system's interface has been designed in such a way as to be as user-friendly as possible, even for people who are not expert computer users, and for this reason reference was made to some of the principles and guidelines of human-computer interaction relating to user-centered design discussed in (Norman, 1986). The interface is based on an icon paradigm: from the implementation viewpoint, this corresponds to the creation of buttons which are often semantically evocative. The icons are distinguished into two categories: those for handling the system, and those making actions possible. In creating the buttons, to guide people in using the system and reducing the risks of confusion often linked to the use of a hypermedial system, use was made of colors, voice and pictures. Color takes on three fundamental functions: it characterizes the entry routes, characterizes the different sectors within the entry routes (for example, the historical development is subdivided into three periods, each marked by a different shade of green), and distinguishes buttons according to their function (e.g. light blue ones allow management of the system). The voice is used to give an instruction or to suggest a cognitive pathway to follow. The pictures provide a visual support for information (maps, or photos or drawings of mechanical devices, etc.). In the system it is also possible to activate dynamic constructions and step-by-step constructions. The former are used to visualize the generation of curves obtained by means of movements of geometrical entities (e.g. two points on two different lines, the quadratix of Dinostratus, etc.), the latter to visualize in successive steps the solving procedure for geometrical problems (e.g. the construction of a segment whose length is the square root of x).

Figures 1 and 2 show some IPER-3 layouts relating to the problem of the quadrature of the circle, in which one can see the various entry routes and some of the functions described above.

EXPERIMENTATION WITH THE SYSTEM

Organization of the experimentation

The first experimentation with the system was carried out with university students in the mathematics degree course who were studying for a specialization in teaching. The experimentation, carried out during the last month of the course, was divided into the following phases:

- Introduction to hypermedia. This phase provided an introduction to hypermedial systems as a means of representing and accessing information. The theoretical basis of such systems was presented; concepts, definitions and explanatory examples referring to their main characteristics were given.
- Presentation of IPER-3. This phase concerned the basic design work, as well as the system's characteristics and use.

- Use of IPER-3 by students. The students were initially assisted in using the system and then left free to explore.
- Writing of a report by the students (individually) on their impressions regarding the use of IPER-3 and the way they had explored the knowledge incorporated in the system, as well as what they had learnt about hypermedial systems.
- Creation of a project (only on paper, not implemented) by the students, divided into groups, to complete the IPER-3 hypermedial system as regards the problems of the duplication of the cube and the trisection of the angle.

We consider the population chosen as suited to this type of experience. Thanks to the course they were attending, the students were wholly used to dealing with mathematical topics and hence it was possible for their attention to focus on the means of transmission and the type of mediation. Furthermore, many of them had attended the course on the history of mathematics.

The considerations made in the pages which follow are based on the students' reports and on their projects for the completion of IPER-3.

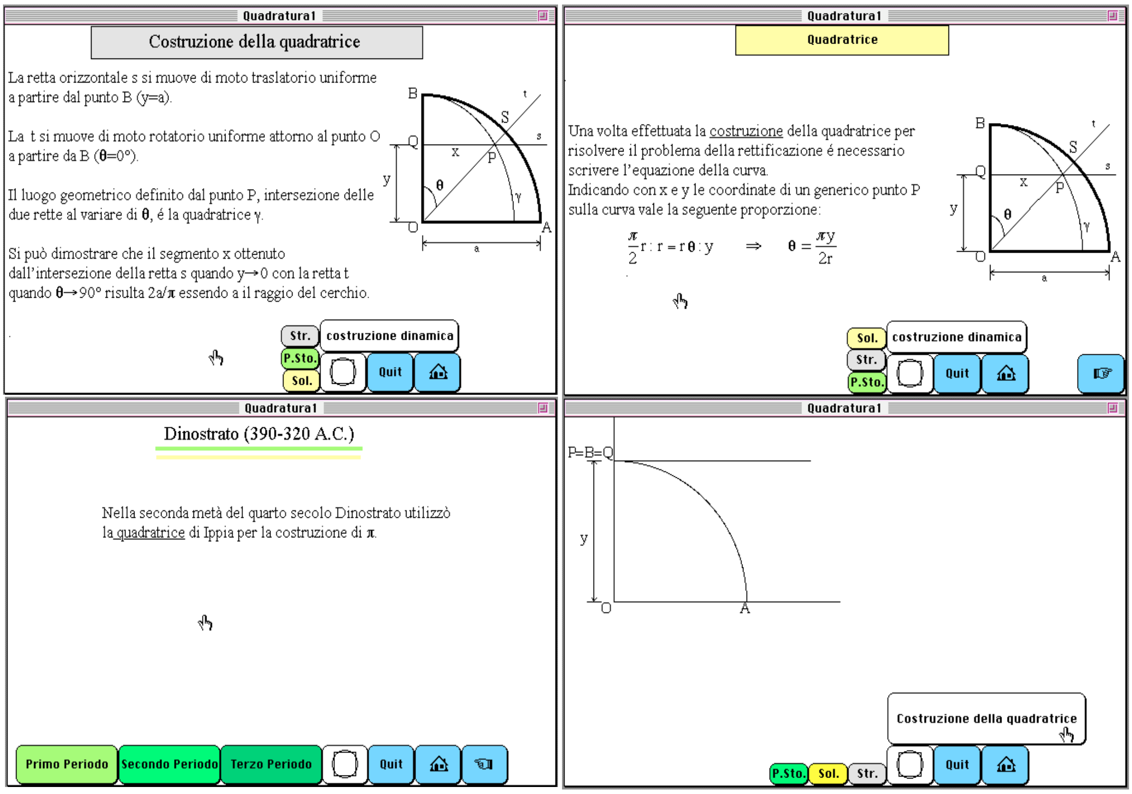

Fig. 1: Four IPER-3 layouts relating to the quadrature of the circle

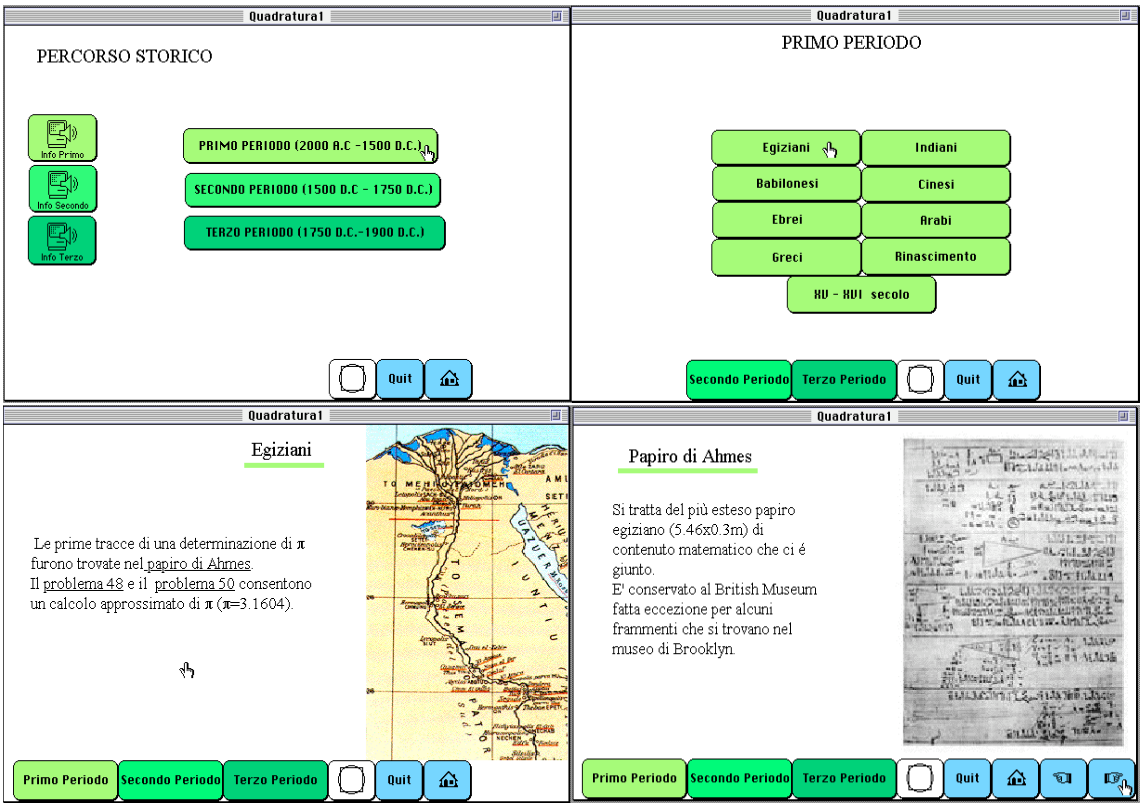

Fig. 2: Four IPER-3 layouts referring to the historical development of the problem of the quadrature of the circle

The students' observations

The students' reports were analyzed by means of a reading grid in which observations were collected in accordance with the following main parameters: observations of a technical nature on the computer and the software, observations on the mathematical contents, observations on hypermedia as learning instruments, observations on their own work and reactions.

Table 1 shows the most significant observations which emerged from the students' reports. In general, we notice curiosity and appreciation in relation to the technological instrument, but also some diffidence regarding its teaching potential. The students show an interest in an integration of resources (instruments for a first approach, for revision, for update, etc.) rather than in replacement of one resource with another.

The students' observations show up a rather generalized need for guidance in the exploration of a new subject; hence the suggestion to include particular features in the interface (some suggest a sound-based guide) to which one can have recourse in case of uncertainty about which way to go.

The request made by many that in the opening pages there should be a statement regarding the impossibility of solving the three classical problems in an elementary fashion, may indicate some resistance towards the 'open' and exploratory approach

which the system would allow. This impression may be linked to the fact that the majority of the students consider the technological instrument as useful for revision, but not for a first introduction to the subject. By contrast, some students emphasize that the possibility of following personal pathways is the aspect they perceived as most positive in the use of IPER-3.
The content chosen is judged very positively in that it makes it possible to deal with mathematical contents in a way which is not static but follows a historical evolution lending itself to different readings (solutions, instruments, epochs) and permits connections between topics often dealt with separately from one another.

Students' projects for completing IPER-3

In the projects made by the students for the completion of IPER-3 we noticed both common tendencies and diversified ones. Both groups exploited certain elements (color, icons) and accepted the IPER-3 idea of distinguishing various approaches to the problem.
The "Trisection of the angle" group with respect to the prototype added the entry route 'curiosity', but created no explicit links between this route and the others. What is judged curious is questionable; however, it is interesting to observe that, thanks to curiosity, the students used the Internet source in addition to books. This source was also exploited to obtain figures, portraits, etc.
The group realized the importance of dynamic constructions and suggested that a user's manual was necessary. In the last part of the project they highlighted the links (and hence they stressed their importance) with some graphic schemata, which in the other groups did not happen.
The "Duplication of the cube" group brought in the new entry route 'demonstrations' which makes navigation more flexible, considering whether one is interested in a given theorem in itself or only in its enunciation in order to go on dealing with a given topic. However, subsequently the group did not make good use of this route with appropriate links, maintaining, by and large, a marked sequentiality in the treatment.

rvations of a technical nature on the computer and software

There is appreciation for the use of different media, e.g. sound.

There is appreciation for the interface and the use made of ‘evocative’ icons; it is seen as useful to always keep the buttons in the same position on the screen and to limit their number.

It is considered useful to be able to keep more than one page open at a given moment with the same interface layout.

• **Observations on the mathematical content**

The history of mathematics is considered an interesting subject; in particular, the importance of being able to see the historical evolution of a mathematical problem is stressed. It is observed that usually mathematics is ‘imposed’ as something finished and bare.

Students observed that it would be possible to utilize IPER-3 with secondary school students, marking out some possible itineraries to follow (e.g.: general information, simple line constructions, rule and compass constructions, demonstration of the transcendence of π).

• **Observations on the way the content is organized in IPER-3**

The need is pointed out to state, in the first phase of the hypertext, the impossibility of solving the three classical problems in an elementary fashion. This is seen as important because the user by himself may not become aware of this impossibility.

According to some, the hypermedia should be constructed after the users have been identified.

The need is stressed for innovative instruments for school updating. Obviously, it is observed that in its present formulation IPER-3 is incomplete for any teacher desiring to use it for updating.

• **Observations on hypermedia as a learning instrument**

It is considered a useful instrument for looking deeper into a given topic and for revision.

By contrast, it is considered a distracting instrument for the first approach to a subject for those who have no knowledge of the contents.

Freedom of navigation is seen as useful, though it is pointed out that there is a need to guide focusing on the main topics.

It is observed that a hypermedial instrument can permit the development of greater mental elasticity, favoring a linkup between different topics.

It is observed that instruments of the type examined can be useful for accustoming students to using other technology-based learning instruments.

There is positive consideration for the opportunity given by hypermedia to connect kindred topics with one another.

• **Students’ observations on their own work and reactions**

A female student observes that she feels the need to be able to construct the geometrical figures on her own, instead of finding figures already drawn (from this point of view step-by-step constructions are positive).

Almost everyone notes the usefulness of the instrument in the revision phase (better than notes); in any case it is an instrument seen as a support to the book.

Some stress that instruments of this type provide an opportunity to approach technology, and to promote a new work mentality.

All express their own curiosity and interest.

Table 1: Overview of students’ observations

PROSPECTS

The system presented is a prototype of a way of constructing a hypermedial system in the history of mathematics. Our way of working with it (in both the construction and utilization phases) can also be considered a prototype and we feel we can make some considerations on it which are transferable to other situations. The points on which we wish to focus our attention are the following:

- working with a hypermedial system can promote *meta-cognition*
- a hypermedial system can be seen as an *environment which lends itself to the detection of 'teaching phenomena'*.

As regards the first point, we must first make some considerations of a general nature. The concept of meta-cognition is present in many theories of education, though with varying interpretations due to the different frameworks (psychology, mathematical education, etc.) and authors. We refer to a recent article (Robert & Robinet, 1996) for an overview of the researches which rotate, in a more or less broad sense, around this concept. We are particularly interested in the approach to meta-cognition, closely linked to practical teaching situations, found in Schoenfeld (1987). In this paper the concept of meta-cognition is introduced explicitly and the author explains that it is necessary to interest the student in knowledge of knowledge ('thinking about thinking'). The author makes a classification of those elements which he considers most important in meta-cognition: knowledge of one's own thought; control of one's own thought in work situations; mental representations (meaning conceptions of mathematics and the way of doing it). Schoenfeld describes four modes of developing meta-cognitive skills in the students: using videotapes on badly organized ways of working in mathematics on the students' part, highlighting the teachers' way of working in mathematics in front of the students (with constant questioning and changes of strategy), organizing discussions with the class in which the teacher has the controlling role, getting the students to work in small groups in which the teacher periodically intervenes on the solving strategies. The final objective of this type of activity is the creation of what he calls a 'microcosm of mathematical culture'.

We can deduce from the analysis of our experience that working with hypermedia serves precisely to promote a new form of meta-cognition in the sense that the students have to reflect on their way of facing the problem, and organize their knowledge and their solution strategies. The idea of reflection on strategies is inherent in the very idea of 'non-random navigation' in the hypermedia. Becoming aware of one's own way of approaching a topic is detected in the phase of discussion on the use of IPER-3, and even more in the project completion phase, in which contents are organized in accordance with conceptual schemata differing from one group to the other (see the different entry routes introduced by the two groups).

The point which considers a hypermedial system as an *environment for the detection of 'teaching phenomena'* is tightly linked to our conception of the mathematics teacher's profession. We are firmly convinced that there are situations in which the teacher must question himself about research on teaching: one of the central issues to be considered is exploration of the students' beliefs and mental schemata. We see this theme as central because we are convinced that pre-existing schemata act as a filter at the moment when a student faces a concept. Hypermedial technologies, in an appropriate use context, can be a means for having information on the students' conceptual networks, since when they work on rendering their strategies explicit they provide the teacher with a key to their mental schemata.

We believe that, over and above the interesting practical aspect linked to easy access to information, seeing the hypermedia as a promoter of meta-cognition and as an environment which lends itself to the detection of 'teaching phenomena' is a significant approach to the analysis of the role that this instrument can play in the teaching/learning of mathematics.

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