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# A Discussion of Judah Schwartz's Chapter

Marcia Linn  
Roy Pea

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*Marcia Linn.* The first question that Roy and I thought might be interesting to discuss is the research paradigm that Judah advocates in his chapter. Judah suggests a 5-step process. The main thrust of the approach is to create an artifact, put it out in schools, and see what happens. One thing I wonder about is the following: Does this approach deny participation in the artifact construction process to those who are ultimately going to use it? Is this approach taken for logistical reasons, or does it have a more philosophical underpinning? In my own experience, with the Computer as Lab Partner Project, I found that there are real advantages to having a joint constructive process involving teachers, researchers, and developers.

*Roy Pea.* There is a general trend in studies of workplace technology, influenced by a group in Denmark that works in participatory design, in which there are close observations of existing work practices. Here, analogous analyses might be concerned with what teachers and students are doing in the algebra classroom, taking a look at the kinds of resources that they have available and what kinds of tools they currently use, and then to engage in a co-design practice with them. Of course, this is intended to complement the good design ideas and instructional goals Judah starts with. Such ideas and goals would not have necessarily come out of conversations with teachers and students.

*Judah Schwartz.* Let me interject, because there is a very interesting issue of balance here. Obviously we do have teachers participating in our design efforts. But there's an interesting conundrum about observations of classroom practice, because if you ask teachers in the main "What do you want?," what you get are requests to support what they know how to do. In algebra, for example, where we are working on the design of some software called the Algebraic Proposers, you get requests for symbol manipulators and graphing packages.

*Marcia Linn.* I think that's absolutely right. Ask people what they think they want, and you get requests for electronic books. Accomplishing an order of magnitude change is clearly a difficult issue. On the other hand, I question whether merely putting an artifact out in classrooms will result in an order of magnitude change. In fact, I think you implied that wasn't necessarily true, that the Geometric Supposers were sometimes used for pretty retrograde purposes.

*Judah Schwartz.* Any tool can be used or abused. For that reason it's very important to get moles in the system. The way to get moles in the system is to use the teachers with whom you've been working in the development process as your front people. I don't talk about the Supposer to teachers. Richard Hood talks about the Supposer. Dan Chazan does; Mary Sapienza does. And that's very important, because then it's talked about to teachers by teachers who have had a hand in the development process. That's a key ingredient in the strategy.

*Roy Pea.* How do you go about establishing attitudinal changes in the teachers you work with? If they're having difficulties of a particular kind, it's not necessarily because they're not learning the technology well, but maybe because the technology could be redesigned in ways that more fit what they come to see as useful work practices with this new tool in their environment.

*Judah Schwartz.* You listen hard and you talk to people as honestly as you can, but you don't take an advocacy position. We work very hard to have the advocates be the teachers we've worked with in the development process. It's not as if the teachers don't talk to us, but we're not the advocates. The ones who are have lived through the process, in classrooms.

*Marcia Linn.* How do you help teachers become experimenters in their own classrooms? Do you feed them a few good ideas, not just about the artifact but about the instructional process, to facilitate a broader instructional impact?

*Judah Schwartz.* You make lots of written materials. For every one of the Supposers there's a thick book of problems and projects.

*Marcia Linn.* And that's not constraining to the open-ended experimental paradigm you advocate?

*Judah Schwartz.* No, because the projects are open-ended, the problems are open-ended.

*Marcia Linn.* There are some serious questions about what instructional lessons students can take from dynamic, interactive media. The Geometric Supposers represent one class of examples, the Function Proposers another. Dynamic representations often flash rapidly across the screen and overload the processing capacity of the student. For example, all of the boxes in the Function Proposers might be difficult to integrate and, instead of leading to a deeper understanding of the relationship between formalisms and functions, might simply lead to confusion. Or, they might spark interest in questions like "Let's see what kind of a picture I can put on the screen," rather than in the prediction and the reconciliation of outcomes. Let me mention some of our experience with the Computer as Lab Partner project. In that project we have real-time data collection in a science class. At first the students were wildly enthusiastic. However, we discovered that the students weren't taking the graph from the screen and relating it to what they thought was going to happen at all. When we made prediction a rigid constraint of the curriculum, it led to an order of magnitude increase in understanding. The difficulty is to integrate the proper use of the dynamic representation with the cognitive excitement it engenders.

*Uri Treisman.* When we talk about this wonderful tool, let's remember that it's only a tool. One has to also develop the settings in which this tool is going to be used. One of the problems with algebra is that students can manipulate quadratics, but they never learn when they would expect to see a quadratic. Or if you ask a teacher to describe a situation in which one might expect a quadratic function to arise, one may or may not get an answer that's productive in classroom terms. We have to help build the contexts of tool use, not just the tools themselves. That raises assessment issues as well. It's difficult to assess beautiful tools without having an understanding of the settings in which students might get to work with them.

*Roy Pea.* I think that's the kind of thing that we've been discussing, having a reciprocal evolution between the technology and the work practices. Indeed, that kind of reciprocal evolution applies as well to the research questions you would want to ask about the classroom environment: What are the kids learning? What does the teacher need to know to be able to pull off activities like this? The research questions that might have been asked by any of us before the Supposers were put into classrooms will change dramatically after we see what goes on in them. Seymour Papert had wonderful ideas about what you can do with Logo in the classroom, but different things happened in the classroom with a lot of teachers. The same will be true of the Supposers, of course. So, we have to keep

questioning the assumptions about what the appropriate layout and functionality of the tool is, what the appropriate research questions are to ask, and what the appropriate work practices and curriculum are. And that leads to the topic of curriculum transformation.

*Judah Schwartz.* There are a number of things to respond to in what's just been said. First, I should stress that we aren't developing all-purpose tools; some tools are good for some things, and others serve different ends. Second, I can at least point to some existent proofs.

I'll tell you one story, because I think it's wonderful. A student of mine who is now teaching in high school is struggling in an undistinguished algebra two class. He's giving the kids functions to classify graphically, lots and lots of pictures of functions. He wants students to classify them, to invent ways of describing them. So the students are classifying them by their asymptotic behavior, by how many zeros they have, how many extrema, and so on. They have some sort of an ill-defined notion of slope. They know what slope means for a straight line, but they don't know what you do with all those wiggly things, where the slope obviously keeps changing. Then one kid says,

"I know, because the slope changes it has to have an  $x$  in it."

He says:

"In fact, I thought about it. Look, suppose you want the slope of [the function  $y = ] x^3$ . If you look closely at  $x^3$ , it looks like a straight line, because everything looks like a straight line if you look closely. So the thing is, you gotta write  $x^3$  the way you write a straight line, which is  $m$  times  $x$  plus  $b$ . So how do you write  $x^3$  like that? You write it  $x^2$  times  $x$  plus  $b$ , and  $x^2$  plays the role of  $m$ ."

That's not bad, not bad at all. That's one of my good stories. To counter that there are a lot of less good stories, of course.

*Alan Schoenfeld.* You said that you want to change the atmosphere in the classroom, to establish a different kind of classroom dynamic that would ultimately affect the students' habits of mind. The classroom reality reflected by your stories is that sometimes it doesn't work, sometimes it does, and there are some wonderful stories to tell when it does. The research question for me is: What makes the magic happen when it happens?

*Judah Schwartz.* I can tell you a little bit about that in the case of geometry. Part of what makes it happen in geometry has nothing to do with the software. It has to do with the organizational surroundings. Some of it has to do with the degree to which a principal is willing to empower teachers and say: "Yes, this is a

good way to teach; yes, this is all right; yes, I will shield you from the school board when they say they have to take the same exams.” (Never mind that when they do take the same exams they do equally well or better.)

In addition, you need to have networks of teachers who can talk to one another and say, “You know what I tried today,” or “You know what some kid came up with today?” They don’t necessarily have to be in the same school.

Those are some of the things that seem to help make the good things happen. They don’t seem to be geometry specific, so when we come to the same point in algebra we will apply the same strategies as a first approximation and hope that works.

*Roy Pea.* One interesting thing about the anecdotes or “war stories” you just referred to is that they could be used in some really interesting and positive ways as resources for the Supposer community. Julian Orr at the Xerox Palo Alto Research Center has done a lot of work on how expertise gets propagated among copier technicians. One of the major ways is not through any of the technical documentation and manuals at all, but through stories that they tell informally at conferences, over drinks, and so forth. In some sense your teacher anecdotes of classroom successes are of the same kind. Maybe one of the things that this shows is that we need a way of rapidly disseminating these positive stories to give teachers a sense of the ways in which the kids and they can be empowered.

*Judah Schwartz.* Indeed. In a somewhat Edwardian fashion, the publisher of the Geometric Supposer has established something called the Geometric Supposer Society. You buy in, as it were, by sending in an anecdote which then gets included in the newsletter. And so this strategy is in fact used.

*Uri Treisman.* I’m not sure that’s enough, and I’m worried about the “mom and apple pie” romanticism that says teachers must be involved, and that magic will happen. In fact, when you look at the teachers who are involved in such projects you find that they are a very special breed. Many of them are actually hired away from their classrooms by universities to do work on such projects.

I think it’s really important to develop a careful understanding of the settings in which teachers learn to use these materials, and what works in them. We need accurate estimates of what the real cost is for teachers to learn about this stuff. Generally, teachers can take anything and turn it into a resource for what they want to do, and that sword cuts both ways.

*Marcia Linn.* That is the point. How do we empower teachers to create an experimenting society in the classroom? It is very difficult. What kind of support is really needed to create the kind of experimenting society where teachers really think they can try out a curriculum, listen to what students say, make some adjustments, and try it again? I think the anecdotes Judah described will help.

But I think that a more collaborative group where teachers could come together and talk to people like Judah would be better. Teachers could say, "Look, this is working, this isn't working," and you could say, "Have you considered this?," or "You are using cooperative groups, but you are encouraging cooperation for an activity where cooperation is very ineffective. Cooperative groups generate ideas well but are less successful at synthesizing ideas." This kind of discussion would move the process along. Materials such as these require changing the entire way one thinks about how teaching takes place, and teachers need support in doing that kind of thinking.

*Andy diSessa.* I wanted to ask a variant of one of the questions we've been discussing. Let me start by saying that I've been trying to think of myself lately as a designer of mediated activities, rather than a designer of artifacts. That change of focus may seem small, but it makes a big difference. Along those lines, I think we ought to be doing more thinking about the structure of activities as kids engage in them. In particular, when you get your good stories, what is the character of the activities that are taking place at that point?

I would like to ask you a question about that, but in order to frame it, I need to explain my mini-theory of the activities that I think you want kids to get engaged in. You want them to engage in a certain kind of mathematical game that we all understand, but it's not clear that a kid has ever participated in something like that. This game goes as follows: In stage number one, you have some domain or artifact, and you kick it a little bit, in ways that you understand you can kick these sorts of things. In stage number two, you notice that there's something interesting in the way it reacted to your kicking. Well, where does that notion of "interesting" come from, and how do you select what's interesting out of what happens? In stage three, and I think maybe this is a critical one, you have to explain the interesting thing. And that's where you invent the mathematics or you discover the mathematics. That's where you ask the question, does this kid have a notion for what is an adequate explanation for this or what kinds of moves to make to explain it. Are you collecting stories that look at kids' versions of "How do I kick this thing," kids' versions of "Oh, that's interesting," and kids' versions of "This is an adequate explanation"?

*Judah Schwartz.* First, I accept your shift from "artifact design" to "design of mediated activities" as a friendly amendment—but with the following reservation. In terms of school culture, I would want to say mediated activities and artifacts, because if you don't focus on artifacts the necessity for artifacts will tend to get lost. I think it's important that there be artifacts, and activities in the world of education do not always imply new artifacts. So I wanted to make sure that I continue to say artifacts, but I completely accept the notion that it's really mediated activities.

Now, about stories. We are indeed collecting stories of the sort you suggest.

We are doing that in algebra, at a very early stage. It is our hope that we can do the same kind of things that we've done in geometry, where we are now capable of building at every level moderately sensible islands of theory about what works, that is, what works in the approach to geometry, what works in the change of classroom behaviors, and what works in the kind of organizational context for the instruction.

*Marcia Linn.* I'm curious as to why you think the artifacts are necessary. Do you see them as a catalyst for change, or as an opportunity to justify change?

*Judah Schwartz.* It's not a philosophical issue, it's a tactical issue. The very existence of the artifact serves as a stimulus for things to happen, and to happen differently.

*Jere Confrey.* I think it is really important to spend some time talking and thinking about design, because I think there is an interesting relationship between the mediated activity and the artifact you're designing. You're not just creating an artifact, you're creating an artifact that's designed to get at particular things. There are certain things that are in essence hardwired into the design of your software, because those are the conceptions that you are instantiating in that piece of software—for example, your conception of what a function is and how you want the software to handle it. Then you want to leave certain things open as possibilities for a wide range of activities when students use the software. So both aspects of design are important for me. My software design needs to be informed by my working very closely with students and trying to imagine what their conception of the thing is that we're working with. It's also really important for me to articulate what my conception of it is, as I design my software. We need to get those issues out in the open as designers and say more than the fact that we're creating artifacts.

*Marcia Linn.* Let's turn to the last topic we wanted to address, which is research questions. You've said that the classroom environments you want to establish are radically different from the classical model of classrooms: patterns of authority will be different, discourse patterns will be different, even the patterns of engagement with mathematics will be different. Supposers will be in a wide variety of classrooms, ranging from those where there are rigid constraints on student behavior to those that have religious nonconstraints on student behavior. That provides a really interesting opportunity to look at the relationships between the kinds of constraints imposed on student interactions and behavior and what develops in those classrooms.

One set of questions centers around the conversations themselves. Which students get involved in the exciting exchanges of information? Which students feel empowered, and which students feel reluctant to enter into the conversations? What are the mediating events that lead to a change in the conversation?

A second set of questions deals with benefits. Who benefits, given that these conversations take place? Often, modeling effective discourse behavior is helpful for other members of the classroom community. Observers of the conversations may profit. Note Roy Pea's comment on work practices.

A third set of questions deals with the establishment of the classroom environment. How do we get teachers to model the kinds of discourse behaviors that we'd like to see, for example? It seems to me that the proposed kind of classroom environments cannot be fully assessed unless these questions are addressed seriously.

*Roy Pea.* We should note that the Supposers are not only Trojan mice for curriculum, but they're also Trojan mice for researchers. For example, the question, "How are we going to assess what students are learning in contexts like these?" takes on a new character in these environments and will force significant changes in assessment methods. We may need measures that look at the kinds of conjectures that students generate. Judah, have you worked on that problem?

*Judah Schwartz.* One thing that environments like the Supposers allow you to do is to move relatively easily from instances to classes of instances. Certainly that's clear in geometry. It's also the case in algebra; when you use the graphical transformation functions, you necessarily build a family of functions. Such families will have one or several invariants. So you can then pose questions not about tokens but about types. You can say "Build me any function that has the following properties." When you ask a question of that form, you're engaging the student in what is fundamentally a design problem, which has a nonunique set of right answers. It seems to me that's a different kind of assessment than we ordinarily perform. These environments lend themselves to the posing of that kind of question, and I think it's a much richer kind of question.

*Marcia Linn.* Following that, you said initially that you wanted to change students' habits of mind. The sense that I got was that you wanted to change them very broadly. It isn't the case that we want them to be poor representations of \$100 graphing calculators, but that we want them to be able to think about problems they might encounter in their everyday life. I was curious as to how you see that generalization gradient getting established.

*Judah Schwartz.* That's too hard for me. . . .

*Marcia Linn.* Well, it really is one of the fundamental questions. . . .

*Judah Schwartz.* I didn't say it wasn't fundamental! Absolutely. I would love to think that new habits of mind acquired in this kind of arena seep out, and this person grows up to be someone who, say, votes intelligently. I would like to think that, but. . .

*Jim Kaput.* We should recognize that there's a difference between the curriculum change you're trying to make now and the kinds of curricular moves that might be possible a decade or two from now. You are talking about change in a system where you assume that kids and teachers are as we have them now. But another angle is to assume that, not long from now, in grades K through 6, kids will spend a tremendous amount of time building functions, modeling situations numerically, finding numerical patterns, drawing pictures, and playing with other models of situations. By middle school they could be pretty fluid at manipulating the models they've built. I'm wondering, given that scenario, what do you think about the role of the kinds of stuff you're advocating here, with respect to graphical manipulations of functions and so on? Where does that fit?

*Judah Schwartz.* I think it would be wonderful if we could postulate that all of the things that should have been done intellectually for and by kids have been done, and that they arrive at the secondary doorstep being intellectually nimble and agile and facile with these things and can use them in rich ways. I would love to believe that. It would be great if that worked so well that the universal notion of function in 6th graders was no longer that of a mapping from one set of numbers to another set of numbers given by an algebraic rule, but something much more general than that. Then I would say this tool has outlived its usefulness—because we could make a much richer tool. But for the moment I think it's a useful tool, because it can provoke all kinds of things that ought to be provoked.

*Uri Treisman.* There's another research issue I'd like to pursue. Tools like the Supposers help us to create settings in which students can develop intimacy with basic objects to the degree that in fact they can have intuitions about what they're doing. How many hundreds of hours do you need to play with numbers to have number sense? How many hundreds of hours do you need to play with geometric objects, or functions, to develop good intuitions about them? How many hundreds of hours of playing around do you really need before what you know is automatic enough and ingrained enough so that you have fodder for creative work and problem solving and design? Because these tools are appealing to children, we may have the opportunity to explore such questions.

*Ron Wenger.* In the short run a large part of the impact of artifacts on the curriculum will be the extent to which they influence the design of traditional textbooks. When we look at current practice from the point of view that the artifact provides us, we see enormous gaps in the existing texts. One of the test cases could be your Geometric Supposer case. Have you seen textbooks or interesting tasks being designed into textbooks, that don't assume the kids have access to software tools, that are qualitatively different from the contents of textbooks before people had that tool?

*Judah Schwartz.* Two-thirds of a yes. There are a lot of manifest influences of the Supposer on new geometry texts—statements like “If you have the Geometric Supposer here’s a problem to try,” with publishers all over the place. Those aren’t always driven by altruistic motives. When a state like Texas says “You gotta link to software,” the response is “Yes, sir, we’ll link to software,” and they put in some due obeisance to software, the Supposer among other things. Those are the bad cases. The good cases are where textbooks really do that; they say, “If you have the Supposer try the following.” It’s much harder to detect the presence of nonspecific references, that is to say, a problem that you think would not have appeared before but now does appear. How would I know such a problem if I saw it? An open conjecture is not likely to have been a problem in many geometry books earlier, and if it appears now, one can speculate that maybe the Supposer had an influence on that. But it’s very hard to detect.

*Ron Wenger.* I don’t think that will be nearly as ambiguous in algebra. I think that there are profoundly different tasks that we could put in books now, which don’t assume a kid has access to these kinds of artifacts, which are qualitatively different.

*Judah Schwartz.* That is absolutely true. When you’ve done some mucking around with graphs, you can begin to ask questions like, “Tell me all the things you can’t change about a cubic, no matter how you scale it. And explain why you believe what you are saying.” That’s an essay question, unlike the kinds of questions that are asked now. And it could be asked even in an environment in which graphing software isn’t available. I think you’re right, that a very good test of duration and depth of the impact of software tools is their impact on printed materials.