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In this paper we report an analysis of a teaching sequence in which Grade 11 students were asked to produce some graphs corresponding to the relationship between time and distance of a cylinder moving up and down an inclined plane. The students were also asked to carry out the experience using a TI 83+ graphic calculator equipped with a sensor, and to discuss and explain the differences between their own graphs and the ones obtained with the calculator. We analyze the students' processes of meaning production in terms of the way diverse semiotic resources such as gestures, graphs, words and artifacts become interwoven during the mathematical activity. Our findings suggest that a complex relationship between gestures and words allow the students to make sense of the time-space graphic expressions.

INTRODUCTION AND THEORETICAL FRAMEWORK

In an artifact-mediated classroom activity, Grade 11 students were asked to investigate and graphically express the relationship between time and distance of a cylinder moving up and down an inclined plane. Strictly speaking, the temporal-spatial *relationship* of the cylinder's motion cannot be *seen*, even if an experiment is materially carried out. Indeed, in such a case, crude perception merely allows one to *see* the cylinder going up and down the inclined plane. Although motions of this kind were only systematically mathematized in the early 17th century, and since then there may be a certain intuition that "traveled distance" and "consumed time" –to use Galileo's words– bear a certain *mathematical relationship*, the graphic expression of such a relationship (which was not conceived until many years after Galileo's work) is certainly much less intuitive. In fact, the temporal-spatial mathematical relationship of a body's motion is an abstract, conceptual and cultural entity. To render this relationship *apparent* in the classroom requires a fine understanding of space, time and movement. In particular, the graphical account of motion may require students having recourse to diverse semiotic resources, such as gestures, words, drawings, coordinate systems, artifacts, etc.

Recent research has shown the cognitive import of gestures, words, and artifacts in the production of graphical as well as algebraic symbolic expressions (Arzarello and Robutti 2001, Roth and Lawless 2002, Robutti and Ferrara 2002, Radford 2002, 2003). The reported research, as well as other research carried out in other scientific fields like linguistics and psychology, indicates that, in the students' talking and gesturing activity, words and gestures play a substantial role, even if the specific role of words and gestures may vary according to the adopted theoretical perspective. For instance, in the early 1980s Kendon contended that gestures express underlying cognitive representations as words supposedly do (Kendon 1981, p. 38). Following this line of thought, McNeill suggested that gestures and speech share the same psychogenetic source (McNeill 1985, see also Crowder 1996). In a more social, interpersonal perspective, gestures and words can be seen as semiotic means that students use to *objectify* knowledge (i.e. to make things and relations *apparent* in their universe of discourse). It is within the latter perspective that the analysis of the students' activity will be conducted in this paper. Considering gestures as (a loose type of) signs, our intention is to investigate how gestures, words, and artifactual actions are mobilized by the students in order to objectify and endow with meaning the emerging mathematical content (i.e. the *referent*) of the sign-graph expressing the conceptual mathematical spatial-temporal relationship of the cylinder motion. More specifically, our goal is to investigate what we want to call "semiotic nodes", that is, pieces of the students' semiotic activity where action, gesture, and word work together to achieve knowledge objectification.

METHODOLOGY

Data Collection: The data presented here come from an ongoing longitudinal classroom-based research program where teaching sequences are elaborated with the teachers. As the research program unfolds, theory, data, and hypothesis are cyclically generated. Usually, in these sequences the students work together in small groups of 3; then the teacher conducts a general discussion allowing the students to expose, confront and discuss their different solutions. In addition to collecting written material, tests and activity sheets, we have three video-cameras filming three groups of students. Subsequently, transcriptions of the video-tapes are produced. These transcriptions allow us to identify salient short passages that are then analyzed in terms of interaction and students' use of semiotic resources. In this paper, we will focus on one of the small groups.

The Teaching Sequence: A two-day mathematical activity based on a hands-on investigation of motion along an inclined plane included different tasks and questions. The instructional design rested on the premise that the mathematical investigation of spatial and temporal relationships in motion problems supposes the cognitive capability of conceptualizing motion from different mathematical reference systems. Bearing this in mind, we will discuss only 3 questions here.

In ***Question 1***, the teacher propelled a cylinder (called cylinder A) upwards, from the bottom of the inclined plane. The students saw the cylinder go up and come down. The students could repeat the experiment as many times as they wished. The teacher then provided the students with an activity sheet and asked them to produce a graph (called graph A) representing the relationship between time and space of cylinder A's motion. The students were given no information concerning the initial point (or point zero) from where the distance should be (qualitatively) measured. We expected the students to locate the point zero on the bottom part of the plane, that is, the point where the cylinder was put in motion (a point that coincides with the body's position). The teacher also asked the students to sketch a second graph (Graph B) in the same coordinate system where graph A was drawn for a *hypothetical* cylinder (cylinder B) put in motion on an identical inclined plane one second after cylinder A started moving (in other words, cylinder A starts at $t_A = 0$ and cylinder B starts at $t_B = 1$). (Cylinder B's motion was hence a "thought experiment").

In ***Question 2***, the students were asked to perform two experiments (motion starting at t=0 and motion starting at t=1 sec) using a TI 83+ calculator and the Calculator Based Ranger (CBR –the motion detector). In the calculator-based experiments, they were instructed to place the CBR at the *top* of the inclined plane. The students, who had previous basic experience with the graph calculator and motion sensor detector, had to compare their graphs A and B to the ones they obtained with the calculator.

Finally, in ***Question 3***, the students were asked to study the graph shown in the right corner of Figure 1. The graph was accompanied by the following instruction: "A group of students drew the following curve to represent the relationship between time and space when a cylinder is propelled upwards on an inclined plane. This group placed the distance origin around the center of the inclined plane. Is this curve correct? Explain in detail your answer."

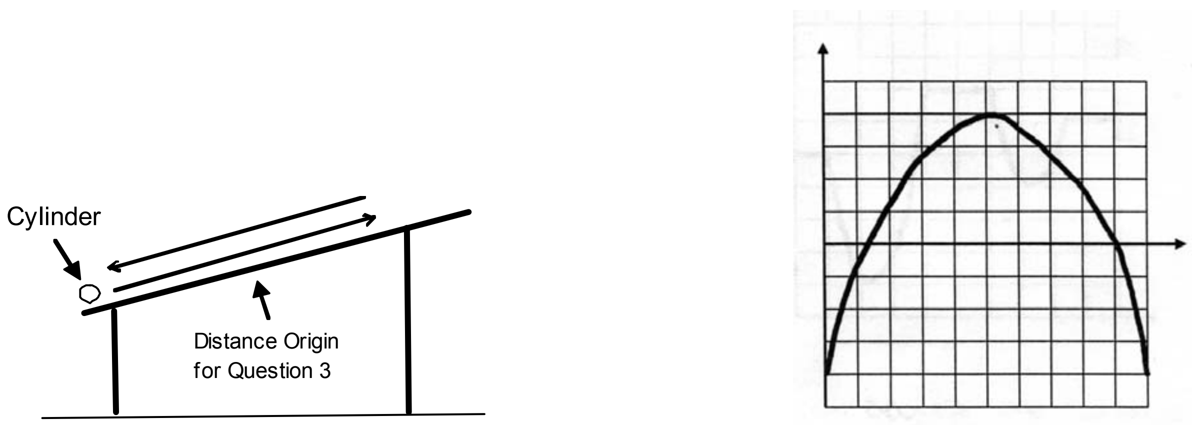

Figure 1. Left: Inclined Plane or Table showing distance origin for Question 3.
Right: Accompanying Graph for Question 3.

RESULTS AND DISCUSSION

Question 1: As expected, the students produced graphs starting at distance D=0 (see *Figure 2*). Key words with which the students gave meaning to the graphs here were the "initial point" (which was equated to point zero of

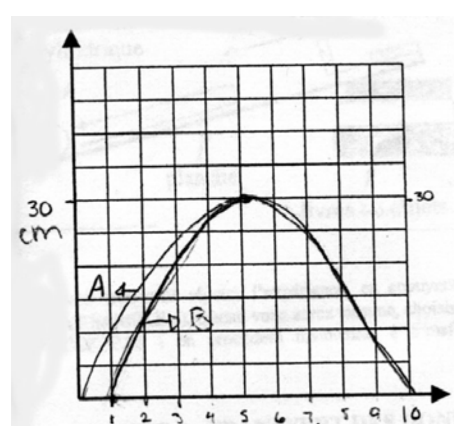

Figure 2. Graphs A and B

traveled space), “going up”, “maximum point”, “going back”.

1. Judith: The initial point is at zero and goes up to the maximum point, then, then …
2. Vanessa (*interrupting*): [it] continues to fall to point zero
3. Judith: (*adding*) while time runs out.

Question 2: As mentioned previously, in this question the students were asked to put the CBR at the *top* of the inclined plane. Let us focus here on the discussion concerning the students' comparison between their delayed motion graph (Graph B in *Fig. 2*) and the calculator's graph (called Graph C and shown in Figure 3).

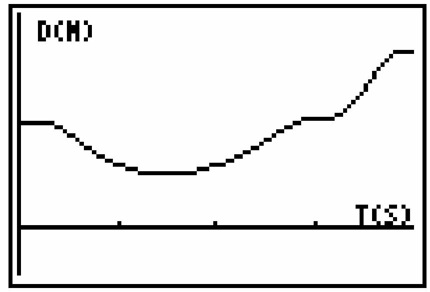

Figure 3. Calculator's Graph C

The students noted several differences, among them the following: (1) Graph C was not perfectly curved in the part after its minimum value, (2) contrary to Graph B, in Graph C the value of the variable D (distance) in the ending points is not the same (i.e. $D_f > D_0$), and (3) Graph B starts at D=0 and its shape is different from Graph C.

<u>Difference (1):</u> This difference was explained by a slight turn of the cylinder when it was rolling upwards on the inclined plane.

<u>Difference (2):</u> This difference was more difficult to understand. After discussing different ideas Judith said:

1. Judith: … (*looking at the inclined plane*) This thing there [the cylinder], does it go further? (*the other two girls turn to see the inclined plane which was behind the students' desks*) … like this … (*she makes a gesture with her right arm; the gesture starts with her arm extended in front of her body and moves back, miming the cylinder motion in its coming back down trajectory*) does it measure the …? Oh!
2. Vanessa: What?
3. Judith: You started on the table [i.e. the table that served as the inclined plane for the experiment], right? (Vanessa : Yes) And when it was rolling it fell off the table (*with a similar gesture her arm is bent again and goes beyond her desk, as the falling cylinder did during the final part of its motion when it fell off the inclined plane and was caught by the student)*... I don't know…
4. Vanessa: It has nothing to do with that.
5. Judith: It does have something to do with that […] That's the curve, right? Here (*she points to the horizontal segment of the left part of Graph C on the calculator screen*) suppose this is when you started on the table and when you finished (*she points now to the horizontal segment of the right part of Graph C*), you've finished further, that's further. […] Let's say that your distance here would be 30, and 45, that's the error! […] Now why it started there (*initial point of Graph C*) … I don't have any clue…

In Lines 1 and 3 Judith makes an “iconic gesture”, that is, a gesture that bears a resemblance with its referent. The iconic sign-gesture *enacts* the falling trajectory of the cylinder. It allows Judith to call her group mates' attention to a specific part of the phenomenon. The iconic gesture affords a segmentation of the phenomenon and operates a choice of what has to be taken into account. Thus, the iconic gesture does not stress speed, time, accurate distance and other elements. What it stresses is the fact that the cylinder went off the table. However, the students mobilized more semiotic resources than gestures. There is, in fact, a coordination of gesture, gaze, and words. Along with gestures, Judith uses locative words and time-related expressions to achieve a coordination of time, space, and movement. This is an example of *semiotic node* (see Figure 4).

In Line 5, Judith has recourse to an "indexical gesture": pointing with her finger, she indicates two parts of the calculator graph on the screen. In this case, numbers (30 cm and 45 cm) come to play the role of the iconic gesture that has previously shown the cylinder falling off the table. The first number represents the students' estimated distance from the cylinder's maximum point to the bottom of the table. However, the cylinder never went 15 cm off the table (i.e. 45-30), for it was caught in the air as it fell off. Numbers are not accurate, and the students do not worry –accuracy is not at stake.

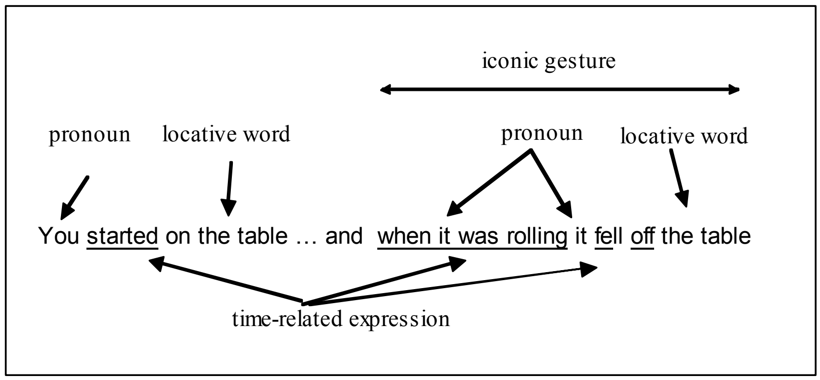

Figure 4. Example of a "semiotic node" where word and gesture achieve a coordination of time, space, and movement.

Difference (3): As the previous excerpt intimates, Judith was able to provide an interpretation for Difference (2), i.e. why Graph C starts and ends at different values of the variable D. Nevertheless, the students' understanding of the relationship between time and distance was still vague. The reference point for the distance remained ambiguous. What the students understood was that the cylinder traveled more distance (absolute distance) in its falling back trajectory than its moving up one. The students kept discussing without success why Graph C does not start at D=0. When the teacher came to see their work, he did not provide an answer. His presence, however, catalyzed the students' ideas, which at the end he reformulated using a metaphor –the "eyes metaphor":

1. Carla: (*talking to the teacher and pointing to the initial point of Graph C*) We don't understand why it didn't start at zero […]
2. Vanessa: It's because it started the other way around, right? […]
3. Teacher: Ah! I don't understand […] we have always rolled the cylinder from the bottom to the top… (*he makes a gesture as if he is rolling up the cylinder*)
4. Carla: (*talking at the same time as the teacher*) Is it because you're further from the thing [i.e. the CBR]?

5. Judith: (*Understanding Carla while the teacher is still talking to Vanessa*) That's true … Ah! Yeah! I get it! It is like we watched the cylinder leave and arrive like this (*she puts her hands on the bottom of the desk*) when it was at the bottom of the table … but now (*she makes a complex gesture: with her left hand placed far from her she signifies the position of the CBR and with her right arm extended and then bending it she mimes the movement of the cylinder coming back to the bottom of the table*) … it's the thing [i.e. the CBR] that is at the top!
6. Vanessa: (*Understanding the other girls*) Ah! Well we weren't looking from the point of view of the thing, it's because of that! O.K.
7. Teacher: O.K. Well there, the point of view … your eyes (*he points to his eyes*) … it's the CBR. For one of the graphs [Graph B] your eyes were at the bottom [of the inclined plane] (*he puts his right hand in front of him and close to his body to signify closeness*) and for one of the graphs [Graph C] your eyes were on … (*he makes a gesture putting his hand in front of him and far from his body to signify the top of the inclined plane*) […]
8. Judith: (*Understanding*) O.K. It's the same thing as that but from a different point of view.

We consider the *gesture-word* systems of Line 5 and Line 7 as two supplementary examples of semiotic nodes. In each case, indeed, a new kind of awareness is made apparent. In Line 5, the semiotic node serves to *make sense* of the fact that D_f is greater than D_0 in Graph C. Epistemologically speaking, this semiotic node has a sense-making constructive dimension. In Line 7, the semiotic node brings to a higher degree of awareness the importance of the position of the spatial origin. It provides the students with a way to better interpret graph motions and to understand what has experimentally happened. Let us now turn to Question 3.

Question 3:

The students remarked that, in the graph, some values of the distance axis "D" are negative. They argued that negative distances are impossible.

1. Judith: No because your distance can't become negative […] It moves away from you or it comes close to you but (*inaudible*).
2. Carla: Well on our graph it does both.
3. Judith: It is because it doesn't go beyond the point? (*the word « point » is accompanied by a gesture of both hands indicating an imaginary point in front of the body*) Let's say that this is zero, zero is here (*she turns her body to the right and places her right hand at the bottom of the right part of her desk to indicate the zero point; there is a coordination of the gesture and the deictic word "here". She is imagining a distance axis having an origin at the bottom of her desk, where her hand is*) and it doesn't go negative because it doesn't go beyond (*she moves her left hand from the top to the bottom of her desk and her left hand goes beyond her right hand that is still signifying the origin. She is implying that, in this reference system, points to the right of this zero point –i.e. points falling beyond the desk– are negative*).
4. Vanessa: I don't know if it is because of that, but what you say makes sense.
5. Carla: (*Carla is not convinced. She interprets the bottom edge of Judith's desk as the horizontal axis of time in their graphs. She says:*) Yeah, but those are the seconds (*after a relatively long pause of approximately 2.5 seconds she waves her hand and draws in the air a concave graphic similar to Graph C while saying:*) On the graph it goes like this … that's the seconds (*she gestures a horizontal line*) … it goes up and comes back down (*she makes again a concave graphic similar to Graph C*)**,** the distance ... (*she makes a vague gesture in the air that tries to locate a position for the distance; she falls silent for a relatively long pause of approximately 3 seconds while she and the other girls think*)

6. Judith: Like your distance starts at zero (*zero is again emphasized using a gesture that indicates a point on the desk close to her. Of course, this assertion is true if the position of the CBR coincides with the body's position)*. […]
7. Carla: Like the first [*Graph A, i.e. the case where the distance is measured from the bottom of the inclined plane*]… the closer it goes to the CBR it will be negative because we started here *(indexical gesture pointing to the actual bottom of the inclined plane*) … [this] started at the top (*the word 'top' refers now to the initial point of the concave graph C that she reproduces here with a right hand gesture*), the lower we were on the x axis … or whatever … the lower we were, the closer we were to the CBR. If we had gone beyond 0.5 m (*i.e. the approximate maximum distance that the cylinder could travel from its maximal position on the inclined plane to the bottom of the table*) it would be negative.
8. Judith: Exactly, it doesn't go beyond the point.

This excerpt stresses the students' difficulties in conceptualizing the difference between the spatial origin of the cylinder motion and the mathematical spatial origin. While the first one was perceptually *seen*, the second, in contrast, requires the students taking into account a theoretical perspective. As Line 1 makes plain, body position provides a powerful perspective ("it moves away *from you* or it comes *close to you*"). But this perspective has to be shifted in order to *make apparent* (or objectify) the phenomenon from other perspectives. Despite the success of the "eyes metaphor" in the previous question, the students could not elaborate a conceptual idea for the point zero distance. In Line 3, Judith mentions the word "point" and accompanies it with an indexical gesture. The concrete point on the bottom of the desk becomes the origin. "Zero is here", she says, and keeps her right hand *there*. Her left arm (initially extended) starts traveling –like the cylinder– from a far position towards the bottom of the desk. And while she is saying that "it doesn't go negative because it doesn't go beyond" her left hand does go beyond the supposed point zero. Here the complex system of iconic and indexical gestures *contradicts* what is uttered. In a sense, Judith is providing us with the enactment of a gestural-and-word-proof by contradiction. And Vanessa finds it meaningful (Line 4). In Line 5, Carla, talking to herself as much as to the other girls, makes an iconic gesture. This time the content of the iconic gesture is not the motion of the traveling cylinder but the calculator-produced graph. Carla's iconic gesture hence has a different referent from Judith's in Line 3. However, in referring to Graph C, the CBR (i.e. the distance origin) should be located at the *top* of the inclined plane. In the following line (Line 6), Judith says that the distance starts at the *bottom*. We see then the students talking about two different origins. The misunderstanding is not clarified. On the contrary, in Line 9, Carla refers to Graph A, switching thereby the origin albeit seemingly without being aware of it. This confusion allows her to interpret Judith's argument and, in the end, consensus is wrongly reached.

Line 3 exhibits another example of *semiotic node*. Line 5 does not. In the latter, the gesture-word system has a heuristic role but it does not produce any novelty in terms of knowledge objectification or meaning production.

CONCLUDING REMARKS

The analysis of the students' semiotic activity carried out in this paper sheds further light on the students' conceptual strategies in understanding motion problems. In our analysis, we paid particular attention to the word and gesture system. Our theoretical construct of *semiotic node* allowed us to locate specific points in the students' semiotic activity where gestures and words achieve a coordination of time, space, and movement leading to the social objectification of abstract mathematical spatial-temporal relationships. The fact that the detected semiotic nodes were strongly oriented to the objectification of the mathematical space origin and the actual motion of the cylinder may explain, to some extent, the students' failure in securing a good mathematical understanding of the problem at hand. Indeed, in these semiotic nodes, time was rarely mathematized. In the students' discussions, time appeared mostly as marking the starting and ending points of the cylinder motion or else it was considered in a very rough qualitative way (as in Line 3 of the students' dialogue related to Question 1 or as in the first example of semiotic node; see Fig. 4). It is true that, in Figure 2, the beginning of Graph B correctly shows the characteristic type of delayed motions, but Graph B ends at the same time as the non-delayed motion Graph A! It may be true, as Koyré (1973) remarks, that it is more difficult to think in terms of time than in terms of space. A suggestion for teaching would be to encourage students to pay due attention to the time variable and to incorporate it in a more sustained way in the analysis of spatial-time relationships.

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