Semiotic mediation for algebra teaching and learning.

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SEMIOTIC MEDIATION FOR ALGEBRA TEACHING AND LEARNING

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In the theoretical framework of Vygotskij's Theory this report discusses the mediating function of particular tools available in a microworld. Following the analysis carried out by one of the authors, in the case of Cabri a new microworld has been set up and a teaching experiment carried out. Some results are reported and the theoretical notion of semiotic mediation discussed.

Introduction

A long-term experiment concerning pupils' introduction to theoretical thinking in the algebra domain has been carried out over the last two years. Activities were accomplished in the field of experience (Boero, 1995) of "symbolic manipulation within a microworld": "L'Algebrista" (Cerulli & Mariotti, 2000) . As in previous experiences, (Mariotti, forthcoming b)), in this experiment the evolution of the field of experience is realized over time through social practices; in particular, classroom verbal interaction is made possible by means of '*mathematical discussion*' (Bartolini Bussi, 1998).

The didactic problem concerns the ways of realizing a theoretical approach to symbolic manipulation. A key-point is that of stating the "system of manipulation rules" as a system of axioms of a theory. The nature of the particular environment may foster the evolution of the theoretical meaning of symbolic manipulation. This is not really the approach pupils are accustomed to, on the contrary, Algebra, and in particular symbolic manipulation, are conceived as sets of unrelated "computing rules", to be memorized and applied.

In this report we shall limit ourselves to the discussion of some elements of the external context (Boero et al., 1995) and their functioning in the concrete realization of classroom activity. These elements may function as instruments of semiotic mediation (Vygotskij, 1978). The general basic hypothesis is that

> *meanings are rooted in the phenomenological experience, but their evolution is achieved by means of social construction in the classroom, under the guidance of the teacher.*

Our research project is aimed at studying the functioning of an artefact, when it is conceived as a potential instrument of semiotic mediation (Vygotskij, 1930-1978). In particular, we focus on a specific type of artefact : a microworld.

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Semiotic mediation

Vygotskij distinguishes between the function of mediation of *technical tools* and that of *psychological tools* (or *signs* or *tools of semiotic mediation*) (Vygotskij, 1978: 53). The use of the term *psychological tools*, referred to signs as internally oriented, is based on the analogy between tools and signs, but also on the relationship which links specific tools and their externally oriented (for the mastering of nature) use to their internal counterpart (for the mastering of oneself) (ibid.: 55).

Through the complex **process of internalisation** (Vygotskij, ibid.), a tool becomes a "psychological tool" and will shape new meanings; in this respect a tool may function as a semiotic mediator.

The following sections will be devoted to illustrating this theoretical hypothesis in the case of a set of particular tools (the microworld L'Algebrista) and the theoretical meaning of algebraic statements, that we shall briefly recall as 'Algebra as a Theory' (in the following AT).

Semiotic mediation in a microworld

A new research project was set up, along the same line and according to the same general hypothesis of a previous research project involving the Cabri microworld (Mariotti et al., 1997; Mariotti & Bartolini, 1998; Mariotti, in press). As far as Cabri is concerned, previous studies have focussed on the analysis of the specific elements of the microworld (dragging facility, commands available, macro …) as an instrument of semiotic mediation that the teacher can use in order to introduce pupils to a theoretical perspective. Taking into account the main results obtained from these studies, a new microworld (L'Algebrista) was designed, incorporating the basic theory of algebraic expressions. Similarly to what happens in Cabri in relation to Geometry, AT, as far as it is imbedded in the microworld, is evoked by the expressions and the commands available in L'Algebrista. As a matter of fact, L'Algebrista is a **symbolic manipulator** which is totally under the user's control: the user can transform expressions on the basis of the commands available; these commands correspond to the fundamental properties of operations, which stand for the **axioms** of a local theory. As a consequence, the activities in the microworld which produce a chain of transformations of one expression into another, correspond to a proof of the equivalence of two expressions in that Theory.

According to the Vygotskian framework, expressions and commands can be thought as external signs of AT and, as such, they may become tools of semiotic mediation (Vygotsky, 1978).

The main elements offered by L'Algebrista as instruments of semiotic mediation related to the theoretic aspects of algebra, are the following:

- *expressions in L'Algebrista* are signs of *algebraic expressions*; their intrinsic structure and their logic within the microworld refer to algebraic expressions as theoretical objects;

- *commands (buttons)* are signs of *axioms* and *definitions of an Algebra Theory*;
- *transforming an expression* into another using the buttons corresponds to *proving* that the two expressions are equivalent, the produced *chain of justified steps* corresponds to a *proof;*
- *new buttons* may be introduced using a specific command (Il Teorematore – i.e. Theorem Maker); such new commands become signs of *theorems*;
- *adding new buttons* to those already available is a sign of the meta-theoretical operation of *adding new theorems* to a theory.

Classroom activities develop constructing a parallel between the activities within the microworld and the evolution of AT. Transformations within L'Algebrista, via the use of the "buttons" available, correspond to proving equivalencies by using axioms in AT. Furthermore, as soon as a new equivalence is derived, i.e. it is proved, it can be introduced as a Theorem, and using the "Theorem Maker" it can become a new "button". The introduction of a new "button" is discussed collectively: thus the evolution of the theory occurs as a social enterprise.

In the following section we shall discuss a number of examples showing some aspects of the internalisation process related to the particular tools offered by the microworld, contributing to the evolution of the theoretical meaning of symbolic transformations.

Making conjectures and proving

The classroom activities consisted in alternated sessions in which the pupils were asked to solve problems inside and outside the microworld. Articulation between these two modalities constitutes one of the main characteristics of the activity design. As far as the analysis of the internalisation process is concerned, activities "without the computer" may highlight some interesting aspects. Let us consider the following problem, to be solved in the paper and pencil environment. Pupils are asked to compare three expressions and to find out which ones are equivalent; they are then requested to prove their conjectures.

Consider the following expressions:

a∗a-b∗b **a∗(a-b)** **(a-b)∗(a+b)**

a) Which of them do you think are equivalent? Which do you think are not? Why? Can you prove this?

b) Analyse your proof and specify, for each step, whether you have used a theorem or an axiom.

Silvio (Fig. 1) starts to reduce the second and third expression into a form that makes it easier to compare them with the first. This part of the protocol looks like typical productions provided by the pupils when asked to *compute* (Italian: “calcolare”). In this case Silvio, although not requested to do so, uses his computing skills to produce a conjecture; nevertheless, his explanation shows that the properties of the operations, i.e. the *axioms* previously introduced, were used in the heuristic phase as *tools* to predict the equivalence.

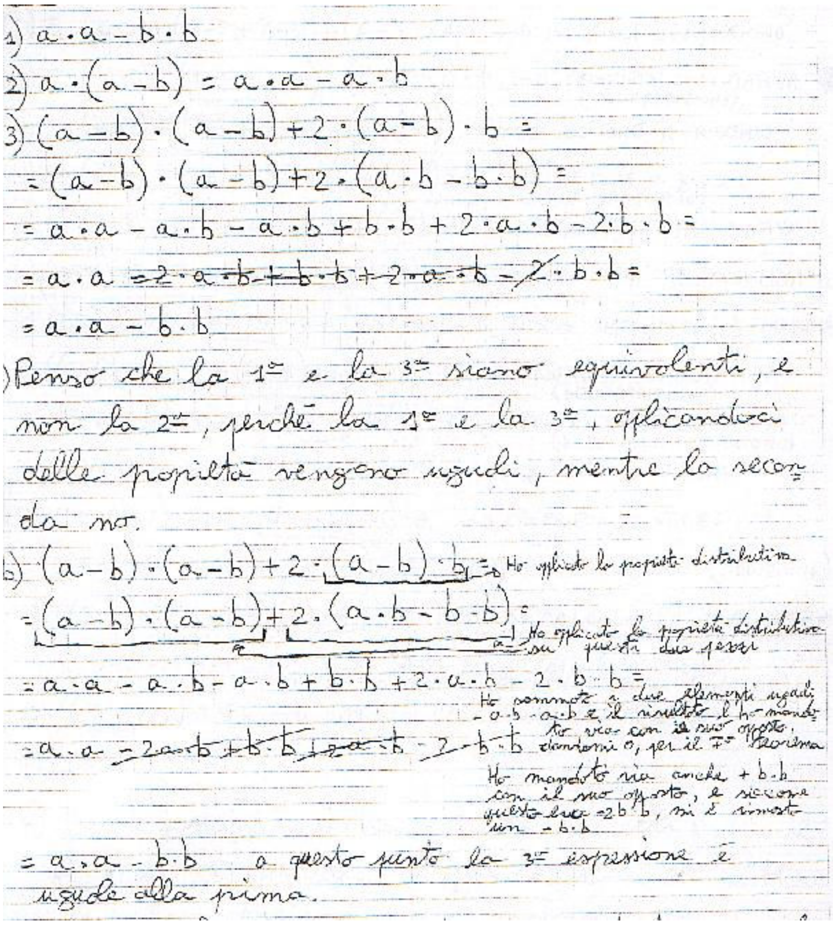

Figure 1. Silvio writes:

I think the 1st and the 3rd are equivalent, but not the 2nd, because applying the properties they become equal, while the 2nd does not.

I applied the distributive property.

I applied the distributive property on these two pieces.

I summed the two equal terms *–a*b* and *–a*b* and I cancelled the result with its opposite obtaining 0 for the 1st theorem.

I cancelled also *+b*b* with its opposite and I obtained *–b*b* because it was **–***2b*b***.**

At this point the 3rd expression is equal to the 1st expression

Unlike Silvio, Marta (Fig. 2) does not use the properties of the operations to produce her conjecture: nevertheless, she produces a correct proof, specifying (as required), at each step, the axiom or the theorem she is applying. According to our hypothesis it looks as though the distinction, clearly defined within the microworld, has been internalised: axioms are represented by given buttons, theorems, once proved are

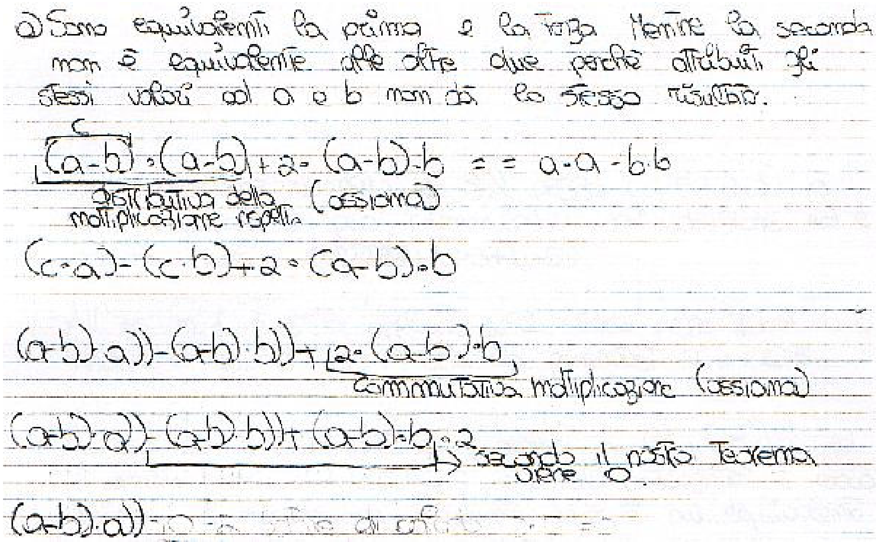

Figure 2. Marta writes:

The first and the third expressions are equivalent, whilst the second is not because giving the same numerical numbers to a and b the result is not the same of the other two.

Distributive property of multiplication (axiom).

Commutate property of multiplication (axiom).

Accoriding to our theorem this is 0

introduced by pupils in the microworld, as new buttons. It is interesting to remark that, consistently with this history of the class community, Marta refers to the theorem, that she is applying, as "our theorem".

Signs derived from L'Algebrista

The basic point of symbolic manipulation concerns the use of operation properties as rules of transformation, which preserve the equivalence between expressions. Although perceived globally, manipulating an expression consists of successively transforming one single chunk at a time; consequently, identifying and treating the structure of an expression becomes fundamental, so as to foresee the effect of single transformations towards a specific global goal.

The representation of an expression within the L'Algebrista incorporates its mathematical tree structure, and this structure becomes explicit, thanks to the *selection function*. As a matter of fact, in order to act on an expression, one first has to select a sub-expression and then to click on a button: the corresponding command will be activated only if the selection is 'correct', i.e. compatible with the property to be applied. The selection function represents an external sign that in the actual interaction with the machine can be used as an external control on one's actions. Internalisation of this control seems to constitute a basic point in the construction of the theoretical meaning of symbolic manipulation.

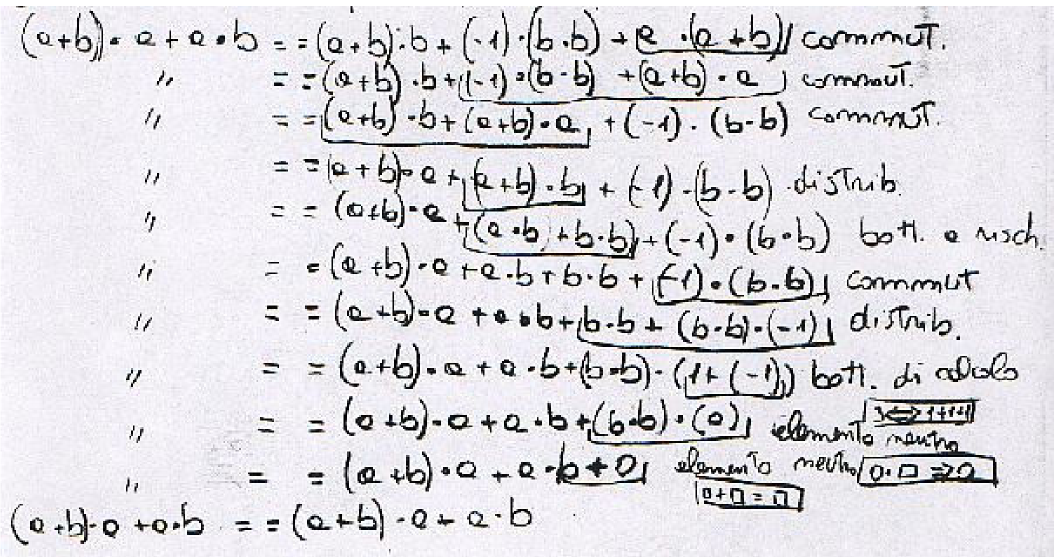

Figure 3. In the case of a comparison task, performed in the paper and pencil environment, the protocol (Fig. 1) shows that pupils use signs clearly derived from L'Algebrista, in particular the *selection function*, or the iconography of the buttons.

Lia's protocol is an example of an interesting phenomenon which can be interpreted as testifying an important phase in the internalisation process. Pupils may use the *selection function* as an external sign controlling the algebraic structure of an expression: Lia tries to prove that the two expressions are equivalent. At each step she underlines (*selects*) a sub-expression and transforms it using an axiom that applies. This behaviour, not explicitly required by the task, refers directly to the interaction between the user and L'Algebrista: Lia refers clearly to the buttons of L'Algebrista using the word *button* (It.: *bottone*) and reproducing the iconography (for instance, in the case of the *buttons of neutral elements* (It.: *elementi neutri*)[1]. At the same time, however, she refers to the properties using the expression commutative, distributive, etc.

Discussion

The instrumental function characteristic of the microworld, functioning as a control on the activity of transformation, is internalised, and contributes to the evolution of the theoretical meaning of symbolic manipulation.

A button provides the external control on the operations accomplished by the machine; the use of the button is interpsychological: it occurs externally, it concerns the interaction between the pupil and the machine; similarly[2] to a personal interchange between human beings, the machine reacts to the subject's action, autonomously from the subject's intention. This reaction may or may not be consistent with the goal of the subject but in any case it directs the activity towards the goal.

The button evolves into a sign and its functioning starts to be oriented internally, so that the button evolves into an intellectual tool regulating the symbolic transformation consistently with the goal of the task. In this respect Silvio's protocol is illuminating. Let us compare the two sequences of computation: according to the Vygotskian explanation, in the activities in the microworld the external sign, i.e. the button to be activated, has realized a break between the steps of automatic computing, allowing the pupil to become aware of the theoretical relationship between two expressions in the chain of computation. The written sign used by Lia and also used by Silvio and Marta is a temporary support control which testifies this evolution. (Similar phenomena of creating temporary sings have been described in Bartolini Bussi et al.1999; Mariotti, forthcoming b)).

A final remark: as already said, our analysis does not take into account a basic point: the role of the teacher in the process of evolution of meanings.

The role of the teacher develops at the meta level, when guiding the evolution of meanings: it becomes determinant in a process of decontestulization required in order

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[1] 0+A ➔A: this button transforms an expression of the kind "0+A" into the expression "A", where "A" can be any expression. This button corresponds to the axiom defining the neutral element of the sum operator.

[2] Similarity with the case of human iteraction should not hide differences, which cannot be ignored. On the contrary, what we are interested in is to study the specificity of a machine, and in particular of computer devices, in the functioning of social construction of meanings. The limits of this paper prevent s full discussion which we are oblige to omit.

to redefine the role of "buttons", and "new buttons", outside the microworld. In fact, commands must be detached from their context and explicitly referred to mathematical theory.

Further investigations into the delicate role played be the teacher are required for a better and clearer description.

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