Randomness and LEGO robots

Michele Cerulli, Augusto Chioccariello, Enrica Lemut

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RANDOMNESS AND LEGO ROBOTS

Cerulli Michele, Chiocchiariello Augusto, Lemut Enrica
Istituto per le Tecnologie Didattiche- CNR di Genova

Abstract
This paper reports on a long term experiment concerning the introduction of 7th grade pupils to the concept of randomness. Pupils are involved in activities with Lego robots, and in the joint enterprise of writing an Encyclopaedia. The main lines of the experiment are provided, together with experimental data, highlighting how some specific elements of the chosen educational approach influenced the evolution of pupils' mastery of the concept of randomness.

1. Introduction

The research we are presenting has been developed in the framework of the Weblabs[1] project, which focuses on "new ways of representing and expressing mathematical and scientific knowledge in european communities of young learners". The teams involved in the project focused on a variety of scientific concepts, developing and testing specific educational approaches based on ad hoc designed technological tools; in particular, our team focused on the concept of "randomness".

The tools used are based on the programming environment ToonTalk (Kahn 2004), and on a computer supported collaborative environment. Moreover, our team was in charge of designing and testing Lego RCX robots, interpreted as advanced technological artefacts embedding knowledge concerning randomness. In a sense, a key assumption is that technological artefacts, such as Lego robots and ToonTalk programs, can be considered as reifications of randomness-related concepts.

In this paper we focus, and discuss, on two main findings concerning the influence of the educational approach employed by us on how pupils learnt about randomness. The first one regards the students' capability to substitute each different random generator in a given physical device; the second one concerns the students' capability to differentiate random from not-random sub-elements in a system.

2. Theoretical framework

What is randomness? What is a random phenomenon? Given a phenomenon how can we judge if it is random or not?

These questions are still open, in the sense that there is not yet a universally accepted definition of randomness. In fact mathematical probability is a quite recent subject, and historians chose 1654 as a convenient landmark for its birth, due to the contents of the correspondence of Pascal and Fermat regarding games of chance. Furthermore its first universally accepted axiomatisation was proposed by Kolmogorov in 1933. Humans have however been coping with randomness for thousands of years, for instance in games of chance, thus it is only its mathematical formalizations that are relatively new. The peculiarity of mathematical formalizations of randomness is that they are based either on common sense, or on key ideas derived from different

[1] We acknowledge the support European Union. Grant IST-2001-32200, for the project "WebLabs: new representational infrastructures for e-learning" (see http://www.weblabs.eu.com/).

scientific contexts. In fact we can find interpretations, and related attempts at formalizations, of the word *random* as: *unpredictable*, *lawless*, *incomputable*, *uncompressible*, *not deterministic*, etc. Any of such characterisation can be ascribed to the idea of randomness, and contributed to define its key aspects, as shown by the historical evolution of the definitions of randomness (Volchan, 2002).
According to this brief historical sketch, it is not surprising that the learning of the concept of randomness (and the related concept of probability) may be difficult, as witnessed by related research literature (Pratt 1998, Wilensky 1993, and Truran 2001). In particular we may focus on the following key educational issues.

Issue 1. A variety pieces of meanings derived from a variety of experiences

The learning of the concept of randomness may be hindered by contrasting views derived from different experiences or from socio-cultural biases. Actually Nisbett (1983) points out the sensitivity of children's response to the situation, as reported by Pratt who comments that "at a low grain size, we see notions of randomness as disconnected pieces of knowledge, with different resources generated by changes in settings". This suggests a need of reflecting on different experiences in order to connect them and build an integrated idea of randomness.

Issue 2. Too much emphasis on determinism can be counter productive in schools

Fischebein's research highlighted how school's emphasis on causality and determinism may have a counter productive result (Fischbein 1975, p.73):

> *"This is why the intuition of chance remains outside of intellectual development, and does not benefit sufficiently from the development of operational schemas of thought, which instead are harnessed solely to the services of deductive reasoning".*

In other words, we can argue that there is a need to put emphasis on indeterminism and randomness, in order to develop intuitions of chance. Moreover, Fischbein suggests: "in order to create new correct probabilistic intuitions the learner must be actively involved in a process of performing chance experiments, of guessing outcomes and evaluating chances, of confronting individual and mass results *a priori* calculated predictions, etc. New correct and powerful probabilistic intuitions cannot be produced by merely practicing probabilistic formulae. The same holds for geometry and for every branch of mathematics." (Fischbein, 1982, p.12).

Issue 3. Needs of theoretical reflection

But, even if certain ad hoc designed experiences may help the development of intuitions, this does not guarantee the development of underlying mathematical ideas and structures, as commented by Pratt (1998, p. 44):

> *"[…] schools might adopt a pedagogy in which children play games in order to experience randomness and build on this informal knowledge, though as I observed in earlier sections such approaches do not necessarily offer a very high chance that the children will attend to the mathematical structures within the game."*

Konold (Konold, 1995, pg. 209) argues that simulations offer us a way of testing our theories, not replacing them, and that theories should remain the primary focus:

> *"My own belief is that this approach has a chance of leaving untouched the informal notions students bring into the classroom. The approach I have used is to encourage students to articulate their informal theories, to make predictions from them, and to use the results of simulation to motivate alternative explanations."*

Konold argues (idbid, p.184) also that:

> *"Typically, people dichotomize, seeing phenomena as "wholly random" .... or as deterministic. .... The kinds of constructions made by the interviewees, the negotiation of meaning for randomness, probability and distributions, are the kinds of bridges necessary to a less dichotomized view."*

These observations suggested to us the need to develop an educational approach based also on pupils' social construction of a knowledge concerning randomness shared by the class. We argue that a useful way towards this goal is to guide and help the pupils, individually and/or as a group, in verbalizing and communicating their evolving knowledge in some steps of the teaching-learning process.

Issue 4 The mediating role of technologies

A wide body of literature exists concerning the mediation role of technologies in relation to the learning of mathematical concepts (Noss & Hoyles, 1996; Bottino 2001). Research such as that conducted by Pratt (2002) and by Wilensky (1993), suggests that microworlds can be fruitfully employed as means for achieving educational goals related to probability. Moreover, Papert suggests a way of empowering the idea of probability by setting up activities that include sample space manipulation, and employing probability (and randomness) as a strategy for problem solving in contexts involving computers and programmable robots (Papert 2000).

Our research is based on the idea of using different microworlds as sources of a variety of meanings that must be integrated in order to build the concept of randomness crucial for understanding probability. We believe that such meanings can be integrated by setting up activities where different microworlds can be compared and connected by focusing on their random aspects. In particular, we use two specific microworlds: the first one is physical and tangible (Lego RCX), the other one (ToonTalk) is virtual and embedded in the computer environment.

3. The Activity Sequence implemented and experienced

3.1. Basic hypotheses

Coherently with the presented theoretical framework, we chose some working hypotheses, functional to the aims of the research. We assumed the importance of:

- developing an investigative atmosphere, giving the students situations to explore;
- focusing pupils' attention and reflection on the distinction between random phenomena and non-random phenomena;
- fostering pupils' capability to assume different standpoints in order to observe, or reflect upon, a given random-related phenomenon, object, or fact;
- pupils' involvement in a variety of experiences involving different kinds of microworlds (in a wide sense), in order to characterize the concept of randomness;

- setting up comparison activities between the different experiences, stressing analogies and differences;

One of the educational aims is that each pupil builds a possible unifying model to be used to describe different random phenomena.

3.2. The design

The designed approach to randomness relies on the exploration of some ***key concepts*** (eg. *predictability*, *unpredictability*, *fairness*, *unfairness*, *determinism*, *indeterminism*, etc.), and of some ***key properties*** of random phenomena (eg. the properties of random walks, the independence of events from their history, etc.). The selected concepts and aspects of randomness are explored in three main phases:

Randomness Small Talks: a collection and analysis of sentences, talks, previous experiences made by the students, directly or indirectly, where the random concept emerges in some way.

Phenomenological approach to randomness: based on the manipulation and reflection on the nature and functioning of ad hoc designed RCX LEGO robots.

Toward mathematization**:** some ad hoc designed computer microworlds, based on ToonTalk, are used to introduce a formal language and mathematical formalization.

In each phase, pupils are required to write individual and collective reports on the activities. In particular the class is engaged in the joint enterprise of building a shared *Encyclopaedia of randomness*. The items of the produced encyclopaedia (and their contents) are derived from the class experiences and from individual and group reports, and are meant to represent the shared culture of the class (Cerulli & Mariotti, 2003). The general methodology is that of negotiating the contents of the encyclopaedia by means of class mathematical discussions (Bartolini Bussi, 1996). Items in the Encyclopaedia are thought of as evolving entities, and in practice they are revised and updated periodically by the class along with the experiments.

3.3. The experimental setting

The experiment is a long term one (2 years, the second of which is in progress), and involves pupils from different european sites participating in several activities for each of the described phases. In this paper we deal only with some activities of the first two phases, which took place in the first year, and concentrate on the data concerning a group of pupils situated in Italy.

We worked with a class of 23 pupils (7^{th} grade, 12-13 year old) in a compulsory school near Milan (Italy). The test has been included in the science and maths curriculum of this class, as set out by local autonomy rules on experimental activity. The class was provided with a portable computer and internet connection, and could occasionally also use 10 computers in the computer laboratory of the school. In total 19 sessions were set up, 13 of which lasted 110 minutes, the remaining ones varied from 25 to 55 minutes, and the last 6 were dedicated to the second phase of the activity sequence. Such a phase consists of several activities involving Lego robots. For each of the 3 employed robots, we set up a session of 110 minutes with practical tasks involving the robot, and a 110 minute session consisting of a class discussion aiming at updating the Randomness Encyclopaedia.

3.4. The context and the submitted tasks

3.4.1. First Phase

In the first phase (called "Randomness Small Talks") pupils are asked to present examples of events related to randomness (Fig.1), and to discuss their random or non-random nature (Fig.1, Task B). Similar activities are then submitted concerning examples of predictable and unpredictable events, and concerning a study of games, proposed by pupils, in terms of randomness and predictability.

Task A: Randomness. Have you ever heard phrases containing the expressions "by chance" or "randomly"? Write these phrases..

Task B: Randomness. We need to agree on the meanings we attribute to the adjectives "random" (or "by chance") and "not random" (or "not by chance")[2]. Write an individual text describing a "random" situation and a "not random" one, use the following schema:

WRITE: examples of "random" situations[3]
INCLUDE: drawings and/or pictures that you find relevant
EXPLAIN: explain why you think such situations are random ones
WRITE: examples of "non random" situations

In class we are going to discuss your texts in order to reach shared meanings for the expressions "random" and "not random".

Fig. 1: The first two tasks submitted to pupils in order to introduce the theme of randomness and to distinguish between random and non-random events. In the Italian text, we use the expressions "*per caso"* and "*a caso"*, respectively for *by chance* and *randomly*.

The first phase ends with a final task in which pupils are required to write a collective class report concerning the meanings of the words "random", "non-random", "predictable", and "unpredictable". They produce the first items of the class Randomness Encyclopaedia, where the contents of the items are socially negotiated and are then structured according to a given template (Fig.2).

Title of encyclopaedia item:
Meanings:
Examples:
Synonyms and contraries:
Related Weblabspaedia items:
Curiosities / Anecdotes / Miscellanea / History:

Fig. 2: Template for Encyclopaedia item.

3.4.2. Second phase

The employed robots have been built by us on an ad hoc basis, and have different levels of transparency, manipulability, and interactivity, as for as their random components are concerned. The first robot that we presented to pupils, the *ShakerBot*, can be driven by a user by means of a special device, the shaker: when the device is shaked, the robot executes a walk, which can be random or not random depending on

[2] The Italian word *casualmente* means either *random* or *by chance*, depending on the context.
[3] The Italian *situazioni*, which we translated with *situations*, stands also for *contexts* and for *facts*.

how the user moves the shaker. In this case the source of randomness consists of the user together with the shaker. In the second robot, the *Drunk Bot*, the source of randomness consists of a mechanical device that is part of the robot, as we will better describe below. In these two robots the devices that are the source of randomness can be easily observed, manipulated and modified, thanks to the properties of their LEGO components. The last robot that we used, the Sweeper Bot, is programmed to move randomly by means of a standard random function which is its source of randomness. In this case its random component is hidden, it is a black box, but it can be used to study the properties of the random walks it produces.

In this paper we focus only on the activities that involved the *Drunk Bot* (Fig. 3). This robot is a vehicle that can execute only two kinds of movements: step forward, and step backward. A special component of the robot, is a random generator (that we called “Roller”), consisting basically of a slide, a pin, a marble, and two sensors (Fig. 3). At each step, the robot “decides” to move backward or forward, according to the sensor hit by the marble in the roller device. In a sense, the robot simulates the walk of a drunk man who is not able to decide whether to go forward or backward. The resulting movement is a one dimensional random walk.

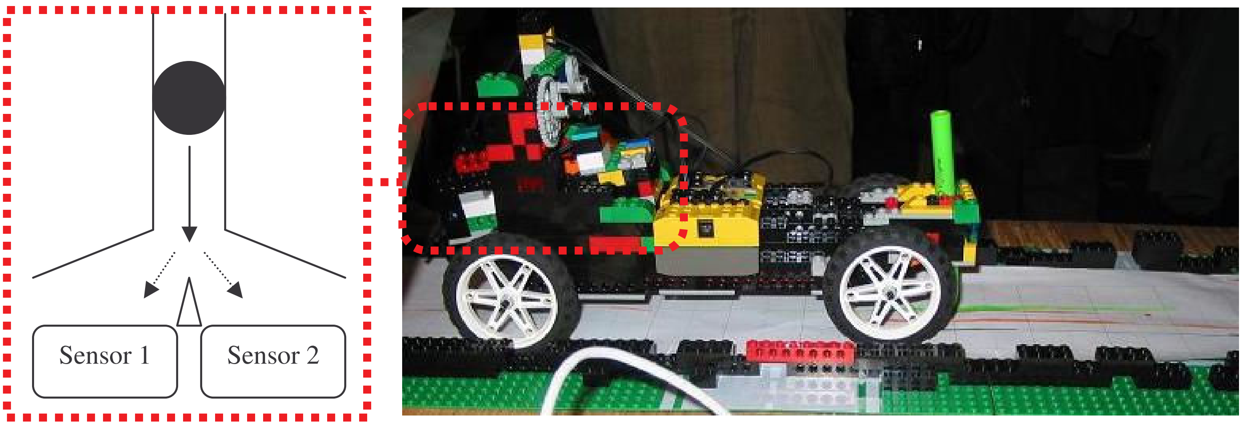

Fig. 3: On the right, the Drunk Bot is free to move on a lane, leaving a coloured trace thanks to a pen. The Roller device consists of a transparent component of the robot which is explained by the left picture. A marble slides down and hits a pin, then it may go left or right (randomly), thus hitting sensor 1 or sensor 2. The Drunk Bot moves a step backward or forward according to the hit sensor.

The task proposed to the pupils requires them to produce and justify conjectures concerning the positions of the robot, after a while. For example: “*where is it going to be?*” ; “*Is it going to be close to, or far from the starting point*?”: “*Does it move forward or backward more*?”. The task is developed in the form of class observations and discussion, the focus of the discussion is guided by the teacher by means of posing questions.

At the end of the second phase, a final Randomness Small Talks is set up, in which pupils are explicitly required to analyse the Lego robots, and to classify them in terms of being random or not random, predictable or unpredictable. The conclusion of the activity is the updating of the Randomness Encyclopaedia. In particular, the teacher brings into class a poster containing all the encyclopaedia items previously developed by the class, and containing photographs of each Lego robot. Pupils are asked to update the poster, indicating, for each LEGO robot, if it can be considered as an example for the actual encyclopaedia items.

4. Results and discussion

In the following we present and discuss some results gathered from the data collected during the Initial Small Talks, and highlighting some aspects of pupils' knowledge related to randomness that evolved throughout the experimentation. Consequently we will show evidence of this evolution, by presenting data from the Final Small Talks, and highlighting how the employed educational approach fostered such evolution.

4.1 Some indications from the initial Randomness Small Talks

In all the examples proposed by pupils the main actor is a human one, and in most of the cases such an actor is the pupil herself/himself. We find for instance pupils proposing examples of random situations such as "*I chose a jacket randomly. (without thinking)*" and "*I found a coin by chance (luck)*". In such examples, the pupil is a constituent part of the considered random phenomenon. In these cases it may be difficult, for the pupil, to assume an external standpoint, which could result in a difficulty in understanding the complexity of random phenomena. As a consequence we believe that there is a need to consider situations where the pupils are not the main actors of the involved random phenomena. A situation of this kind is suggested by the following example proposed by Ciufciuf (one of our pupils): "We are chosen randomly to be examined". In this case the main actor is "the teacher" who participates in the "random" phenomenon of choosing the pupil to be examined. The difficulties of changing stand point is demonstrated by the following excerpt of a text written by a pupil (Vale) reporting a class discussion concerning the random nature of the considered situation:

> *"Ciufciuf said that for us pupils the sentence could be random, because we don't know who will be chosen for interrogation, while for the teacher it is not random because she can decide who she is going to interrogate. […] Not everyone was convinced so the teacher asked us to elaborate with other examples…"*.

Ciufciuf attempts to analyse the phenomenon by assuming two different standpoints, but this attitude remains isolated and the rest of the class does not follow his position. Here we observe that at each step of the proposed activity sequence, pupils are required to discuss the nature of the considered phenomena, trying to reach a shared position in terms of classifying the phenomena as random or non random.

4.2 Some indications from the final Randomness Small Talks:

In this part of our experimentation the employed Lego RCX robots were pre-built tools, whose peculiar characteristic was their "transparency" for the users. This transparency allowed pupils to investigate the different components of the robots, and their specific functions, providing a rich source for reflecting on randomness, as shown by the examples provided in what follows.

4.2.1. Is the Drunk Bot random or not?

During the final Randomness Small Talks, pupils are asked to discuss the random/non-random nature of the Lego robots, in order to reach an agreement to be expressed in the form of encyclopaedia item. In particular they discuss the random/non-random nature of the Drunk Bot. In the following we are going to analyse some key steps of such class a discussion.

4.2.2. Step 1 - The Drunk Bot is not random!

The episode begins with the teacher asking pupils to express their opinion

1. T: *What about the drunk one? (meaning "is it random or not?")*

5. C1: *so, the drunk one, from our point of view moves randomly, but from his point of view he does not…does not go randomly…*

6. *Many voices, we can hear many different opinions!*

First of all, we observe that C1 seems to be able to judge the situation changing standpoint. In fact she talks both of "our point of view" and "the robot's point of view". Such a shift of standpoint, enables her to question the nature of the drunk bot assuming a position which starts a rich and meaningful discussion among pupils, that lasts about 15 minutes, in which different opinions are expressed, and the functioning of the robot is discussed. Below we highlight some interesting passages.

4.2.3. Step 2 - The Drunk Bot is like a special elevator

In order to clarify her position, and convince her pals, C1 presents an interesting example:

112. C1: *The Drunk Bot is like a sort of elevator where there are 100 buttons, but we do not know to which floor each button corresponds [...] and we just push a random button.*

114. C1: *for me it is random, because….one button is like any other, but it is not random for the elevator because it knows which floor to go.*

C1 is comparing the Drunk Bot with a special elevator, with no inscription on the buttons, such an elevator moves randomly from the point of view of a user, but from its point of view it does not move random. However, such an explanation is not enough to convince C1's friends, and the discussion goes on.

4.2.4. Step 3 - Using different random generators

We observe that C1 associates a random phenomenon, related to the Drunk bot, to another random phenomenon, related to an elevator, showing an ability to connect and compare different random generators. This we believe to be a positive result, because literature on the subject had shown that pupils may find difficulties in interpreting different random phenomena as all representing randomness. Rather they may tend to interpret them as totally disconnected phenomena.

We found some more data on this issue. In fact one of the pupils recalls a special situation in which the class substituted, with a coin, the special random generator of the Drunk Bot. The movements of the Robot were still the same then, but the direction to be taken was chosen by means of throwing a coin, instead of using the Roller system of the robot, which depends on the movements of a marble.

136. C2: *what about when we used the coin?*

137. C1: *it [Drunk Bot] moved randomly!*

This excerpt witnesses again the pupils ability to make connections between different random phenomena, moreover it suggests to us that the study of a unique random phenomenon (the movements of the Drunk Bot), which is driven by different random generators (either the coin or the Roller, or other system) can help pupils to interpret different random generators under the same idea of randomness. In other words, we start from different random generators, and we use them as interchangeable parts of unique random phenomenon, this provides pupils with a natural link connection between the different random generators.

4.2.5. Step 4 - The Drunk Bot is a mixed thing

The discussion started by C1 ends up with a pupil, C3 clarifying C1's ideas:

166. C3: *[...] C1 means to say that [...] where the ball goes is random, while the movement done by the robot is not random, but however it is dictated by the movement of the ball, which is random*

167. C3: *it is a random thing that we move non randomly*

168. C4: *it is a mixed thing*

In other words pupils are able to distinguish which element of the Drunk Bot are random and which are not, they are able to decompose the phenomenon into a random part and into a non random part, which we again consider to be a meaningful result in terms of the ability to individuate randomness in given phenomena.

5. Conclusions

The analysed data suggests that the ability of changing standpoints and also taking external standpoints, can give insights into the complexity of random phenomena. In particular it may allow the pupil to individuate the random and non random components of a complex phenomena on the one hand, and on the other hand to compare different phenomena by comparing their random components. We believe that the attitude, and capability, to consider different standpoints, can be fostered by proposing pupils activities involving physical microworlds, which are external from the pupil allowing a detachment from the phenomenon.

The second key indication we abstracted from the data is derived by observing that pupils actually individuated the random generator of the Drunk Bot, and hypothetically substituted it with another random generator. Such substitution was functional to the ongoing class discussion aimed at classifying the drunk bot in terms of being random or non random. The pupils conclude the discussion agreeing on considering the robot as a mixed entity, both random and non-random. In this passage we believe that a key role was played on the one hand by the request of classifying the robot, and on the other hand by the design rational underlying the random phenomena proposed in the activity sequence. In fact each proposed phenomenon has a random generator which some how dictates the behaviour of the other parts which are not actually random, as clearly explained by C1 in the reported class discussion. In this perspective, the random generator of a phenomenon, can be "taken out" and substituted with another random generator, taken from another phenomenon, as in the case of the coin used to "drive" the Drunk Bot. If that is the case, we argue that the fact that the two different random generators are employed as equivalent random components dictating a complex phenomenon, may foster the building of connections between the meanings raising from the study of each of the two random generators. We plan to test this hypothesis in the rest of our experimentation which will be based on computer microworlds that will be designed ad hoc following the principles we presented in this paper.

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