In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

HIDING AND SHOWING CONSTRUCTION ELEMENTS IN A DYNAMIC GEOMETRY SOFTWARE: A FOCUSING PROCESS

Federica Olivero, Graduate School of Education, University of Bristol UK

This paper draws on a study investigating the use dynamic geometry software in the context of open geometry problems requiring conjecturing and proving at secondary school level. After setting the context and main result of the study, the paper will focus in particular on the analysis of the hide/show tool available in Cabri. The way students exploit the possibility of hiding and showing the construction elements of a configuration at stake was revealed to play a fundamental role in the development of the proving process. This will be illustrated through examples from students' work and implications for teaching will be drawn.

INTRODUCTION: PROVING AS A FOCUSING PROCESS

This paper focuses on a particular aspect of a study (Olivero, 2002) investigating the use of dynamic geometry software in the context of solving open problems in geometry that require conjecturing and proving.

The study showed that the proving process[1] within a dynamic geometry environment can be described as a progressive *focusing process*, in which new empirical and theoretical elements (figures, statements and relationships among them, theoretical properties) emerge and are transformed over time by the students towards the construction of conjectures and proofs. The *focusing process* requires what Godfrey refers to as "developing a *geometrical eye*" which he defines as "the power of seeing geometrical properties detach themselves from a figure" (Godfrey, 1910, p.197). Fujita & Jones (2002) illustrate the idea of geometrical eye with an example. Consider the problem: if A and B are the midpoints of the equal sides XY and XZ of an isosceles triangle, prove that AZ=BY (Figure 1). In order to be able to prove this, one needs to 'see' first of all that, for example, triangles AYZ and BZY are likely to be congruent.

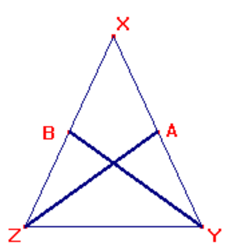

Figure 1. Developing a *geometrical eye*

A key element of the proving process is to develop the capacity of focusing on the appropriate objects at the appropriate time in the process and being able to change focus whenever needed, whenever new elements are discovered and whenever new

[1] The proving process is defined as the process of exploring a situation, formulating a conjecture and constructing a proof (Olivero, 2002, p.41).

In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

theoretical elements emerge. In the previous example one needs to '*see* two triangles as congruent', i.e. triangles AYZ and BZY need to become the object of the focusing process and the property of being congruent needs to "detach" itself from the figure.

A condition that can help the *focusing process* is the possibility of having a field of experience which allows students to manipulate, interact, and change the objects they deal with: such an empirical experience is likely to evoke theoretical elements. The research this paper draws on, showed that open problems (Arsac *et al.*, 1988) and the dynamic geometry environment support this process.

METHODOLOGY

The study consisted of classroom interventions which took place in three secondary schools (15-17 years old pupils) in England and Italy. Students were asked to solve open geometry problems involving conjecturing and proving, working in pairs and using Cabri. Through an in-depth analysis of case studies of six pairs of students[2], an analytical and explanatory framework, that identifies the key elements in the development of the proving process with respect to the affordances offered by the dynamic geometry environment, was developed[3]. This paper examines in particular the role of the hide/show function in Cabri as a tool to support the *focusing process*.

THE HIDE/SHOW TOOL IN CABRI

Most dynamic geometry software offers the possibility of hiding elements of a figure after it has been constructed, and then showing back any of those hidden elements as required. 'Hiding', which differs from 'deleting' an element completely, is a feature that is not available in paper and pencil[4]. As we can see from Figure 2, hiding or showing elements of a configuration at stake changes the nature of the figure to explore because what is visible changes and therefore the potential elements of the focusing process change too.

Figure 2. Hiding and showing construction elements in the problem 'Perpendicular bisectors of a quadrilateral'.

In the context of the study referred to in this paper, the research problem tackled is: how does the use of the hide/show function affect the proving process and the way

[2] The six pairs were video-recorded and observed during the classroom sessions and their Cabri files collected. The results reported in this paper draw on the analysis of all pairs.

[3] For more details on the study see (Olivero, 2002).

[4] Hidden elements are still there but simply temporarily not visible. In paper & pencil the only option available is to delete an element by rubbing it out.

In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

the focusing develops? The study considered in particular the hiding and showing of construction elements, i.e. the elements that link a basic figure with a figure that is dependent on it; for example, in the problem 'Perpendicular bisectors of a quadrilateral'[5] the construction elements are the perpendicular bisectors and the basic objects are the sides of the initial quadrilateral (or the quadrilateral itself).

Three clearly different ways of working with construction elements appeared in the students' proving processes:

1. a systematic use of the hide/show tool: hiding construction elements when exploring and showing them when proving;
2. leaving construction elements always visible;
3. hiding construction elements from some point of the conjecturing onwards and not showing them again.

In the following sections the way the hide/show tool shapes the development of the proving process will be illustrated through two particular examples, which show modalities 1. and 2.. The 15-year-old Italian students are solving the problem 'The perpendicular bisectors of a quadrilateral' in pairs.

A SYSTEMATIC USE OF THE HIDE/SHOW TOOL

This example shows a very systematic way of hiding and showing construction elements (the perpendicular bisectors in this case): throughout the whole proving process, Bartolomeo and Tiziana hide the construction lines when exploring and they make them visible when proving, moving between the two configurations in Figure 2.

The students hide the construction lines straight after finishing the construction before starting the exploration with dragging, leaving only the two quadrilaterals ABCD and HKLM visible.

55 Bartolomeo: **delete** the lines, the points are connected anyway.

While saying this, they transform ABCD from the configuration on the left to the configuration on the right in Figure 2. The perpendicular bisectors are no longer needed as "the points are connected anyway": the perpendicular bisectors are seen as a tool to construct HKLM and once this is constructed they can be 'deleted'[6]. The bisectors are always hidden when they continue with the exploration process.

198 Bartolomeo: so we need to look at the rhombus.

199 Bartolomeo **hides** the perpendicular bisectors and then drags A, D and B.

The perpendicular bisectors are made visible again every time the students attempt to prove something.

[5] You are given a quadrilateral ABCD. Construct the perpendicular bisectors of its sides: a of AB, b of BC, c of CD, d of DA. H is the intersection point of a and b, K of a and d, L of c and d, M of c and b. Investigate how HKLM changes in relation to ABCD. Prove your conjectures.

[6] Hidden.

In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

233 Bartolomeo: here you are … so another trapezium is formed. Let's **prove** it. So let's **put the perpendicular bisectors again**.

The modality of using the hide/show function shown by Bartolomeo and Tiziana has an impact on the perception of the figures on the screen and the development of the proving process. Hiding the construction lines allows isolating the two quadrilaterals and therefore induces the formulation of conjectures on the relationships between the shapes of the two quadrilaterals ABCD and HKLM (for example: "If ABCD is a parallelogram then this HKLM is a parallelogram too"). The conjectures and corresponding figures are then transformed for the proof, by stating the property that will be proven (for example: "so … we must prove that those two [perpendicular bisectors] are parallel") and by restoring the construction lines.

However, it has been observed that showing the construction lines is not always sufficient to recall all the properties that were used in the construction itself, as shown by the extract below.

175 Bartolomeo: so, wait, **if this is a right angle** I can say that…(Figure 3)

176 Tiziana: no, **but it's not** a right angle, what are you talking about? So

177 Bartolomeo: … it must be, otherwise they are not parallel […]

192 Bartolomeo: … so… let's do this… but look, here there are four right angles, otherwise they are not parallel

193 Tiziana: oh dear, **look!… the perpendicular bisector**, isn't it? (she points at b) There is always a right angle!

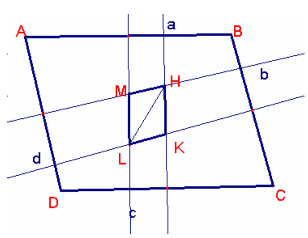

Figure 3. Proving the conjecture "If ABCD is a parallelogram then HKLM is a parallelogram too"

While proving the case of the parallelogram, Bartolomeo and Tiziana do not pay attention to the fact that the lines they made visible again (the perpendicular bisectors), are actually perpendicular bisectors, i.e. lines perpendicular to the sides of ABCD. They spend time in constructing a proof in which something is always missing, that is a right angle (175-176), which is there but is not 'seen' until the very end of the proof (193).

LEAVING CONSTRUCTION LINES ALWAYS VISIBLE

This example shows a case in which the students leave the construction lines visible at all times during the proving process. The extract below shows how this affects Debora and Giulia's formulation of conjectures and attempt to prove a conjecture which is based on empirical elements, i.e. on what they see on the screen only, and does not bring in any theoretical element.

In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

189 Debora: this is congruent to this (Ac'Mb' and Kd'Ca')[7], this is congruent to this (aBbH and DcLd) this is congruent to this (Hbd'K and MLdb') this is congruent to this (c'aHM and LKa'c) [looking at Figure 4Figure 4]

190 Giulia: **the figures internal to the quadrilateral, excluding HKLM, are congruent in pairs respectively**

191 Debora: the opposite are congruent […]

234 Debora: I'm trying to understand from which point to look at it .. if this one …then it becomes something like that. I can't understand which are the biggest sides …ah, it's upside down! […]

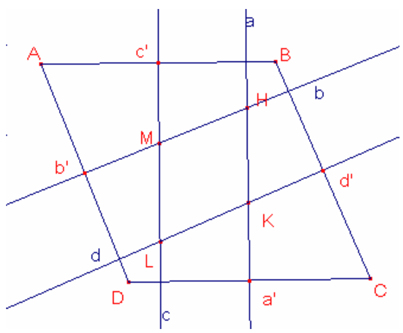

Figure 4. Showing construction elements

The students' exploration up to this moment in the process happens within the spatio-graphical field (Laborde, 2004) as Debora and Giulia are trying to 'read' the figure and the statements they produce are descriptions of facts which can be observed on the Cabri figure (189-191). We can see that the fact that the construction lines are visible has an impact on the conjecturing process: the students focus the attention on the small parts in which ABCD is divided by the perpendicular bisectors rather than on the two quadrilaterals ABCD and HKLM. Figure 4, in which all construction lines are visible, shows both how the multitude of the small quadrilaterals in which ABCD is divided and the possible congruencies amongst them may capture the attention and how difficult it is to 'see' the two quadrilaterals ABCD and HKLM and the relationship between them.

442 Giulia: **so, proofs**. We must prove it is an upside down rhombus …here is the story … the congruence stuff. Let's start from these two big figures: this one (AaHb') and this one (Ld'Cc). Can you see them? So, let's prove that this one (c'aHM) equals this one (LKa'c), and that this bit (Ac'Mb') equals this bit (Kd'Ca').

Line 442 is the starting point of the proof for conjecture 'If ABCD rhombus then HKLM rhombus'. As we can see, what Debora and Giulia want to prove is what they focused on in the exploration (as shown in the previous extract), that is the congruence of the quadrilaterals formed inside ABCD and external to HKLM, due to

---

7 a, b, c, d, a', b', c', d' are used by the students to indicate the intersection points of the perpendicular bisectors with the sides of ABCD (see Figure 4).

In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

the fact that the exploration has been led by the fact that the construction lines are visible, with no apparent theoretical control over what these lines are. This focus does not lead them anywhere, and they remain at a spatio-graphical level for a long time, without succeeding in constructing a proof.

HIDE/SHOW AS A FOCUSING TOOL

The analysis of students' protocols (Olivero, 2002) has shown that during the proving process, the tools available in Cabri (dragging, measuring, hide/show etc) become *tools for focusing* that can be used by the students to shape the way the focusing takes place. The hide/show function can be seen as a *focusing tool* in itself because the possibility of showing or hiding elements allows focusing on different objects/properties. When the construction lines are hidden, then the exploration takes place at a more visual level and theoretical elements do not always play a role or emerge in that process. In this case students do not usually pay attention to the construction and therefore to the geometrical link between the two quadrilaterals at stake. Sometimes, the construction elements may be ignored, and the geometrical properties necessary for proving may not be used because they are/were 'hidden', which is what happened in the case of Bartolomeo and Tiziana analysed above.

When the construction lines are visible, then the geometrical link between the two quadrilaterals is explicitly visible and in general the exploration already contains some justification elements (Olivero, 2002). The links between ABCD and HKLM are seen not only globally, i.e. in terms of quadrilaterals as wholes, but also, and particularly, locally, i.e. in terms of properties of specific elements of the figure (e.g. sides or angles). For example, another pair of students formulated conjectures about the relationship between two pairs of opposite sides ("so whenever the outside lines [sides of ABCD] are parallel the inside ones [sides of HKLM] are"), rather than about ABCD and HKLM. The fact that the parallelism of the sides of ABCD implies the parallelism of the sides of HKLM is what these students prove later in the process. When the construction lines are visible, the situation seems to require a stronger theoretical control over the figure as there are more elements that need to be appropriately managed at the same time. For example, as shown in the previous section, for Debora and Giulia the fact of having the perpendicular bisectors visible has a negative effect: once they stop dragging, the students are not able to distinguish parameters and variables, and consequently hypothesis and thesis. All lines seem to have the same status, so that what they 'see' on the screen is a figure split into many small quadrilaterals by the perpendicular bisectors rather then two quadrilaterals linked through the perpendicular bisectors. Hölzl (2001) deals with similar issues and suggests that we need to find ways to help students focus on invariants rather then focus on details which suppress the overall. In other words, there is the need to develop a *geometrical eye* that *sees* and *focuses on* only what is relevant.

There is a strong link between what one sees and what one uses in constructing a proof. By allowing students to decide what to leave visible and what to hide, the

In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Prague, Czech Republic (2006)

hide/show tool gives students control over the theoretical elements they want to use. This raises questions related to how the theory is/can be made explicit during the proving process. As other research has shown, constructing geometric figures in Cabri fosters theoretical thinking (Mariotti, 2000). In fact, in order to construct a figure in Cabri, the geometric properties of that figure are needed for the construction itself, while this does not necessarily happen on paper[8]. However, the situation is different when some constructions are required on a general quadrilateral (e.g. constructing the quadrilateral formed by the intersection of the perpendicular bisectors of a given quadrilateral). In Cabri, the fact that there is a menu command[9] that constructs the perpendicular bisector of a segment, allows the students to use it without thinking about the property of the perpendicular bisector with respect to the segment. The only thing to do is to find and use the corresponding command. On the contrary, if they were constructing perpendicular bisectors with pencil and paper they would need to think about how to draw them, i.e. they would need to know that they are perpendicular to the side and go through its midpoint. Therefore in this type of problems in Cabri the geometric properties are not needed at the beginning and potentially are not evoked[10]. This would explain why some students seem not to pay attention to the properties of the construction (e.g. the fact that perpendicular bisectors are perpendicular to a side) while proving, as in the case of Bartolomeo and Tiziana reported above.

CONCLUSIONS AND IMPLICATIONS FOR TEACHING

The possibility of hiding and showing elements in Cabri is a 'new' powerful tool of dynamic geometry software, because according to what is left visible the focus can shift to different elements. What students see on the screen influences the construction of conjectures and proofs and choosing what they want to see on the screen influences the proving process. In Cabri pupils are in control of what is on the screen in an interactive way and they can adjust the situation by hiding and showing elements to deal with new discoveries or ideas.

The hide/show tool can be interpreted from a teaching perspective and should become object of teaching. It is important that teachers are aware of this tool (together with the other Cabri tools[11]) and that they make it explicit to students as well, so that they are introduced to a use of Cabri, which helps the focusing process, i.e. Cabri is transformed into an appropriate instrument that is then internalised (Mariotti, 2002). Showing construction lines, together with dragging the figure, will help the students to keep in mind the properties of the construction. Hiding some elements may be useful when wanting to focus on some particular configuration, for

---

[8] For example, when drawing a square on paper the properties of having equal sides and right angles do not necessarily need to be evoked; while if a square is drawn in Cabri by only reproducing a mental image associated with that particular name, without using its properties, then that figure will be messed up when dragging it.

[9] Which embeds a macro-construction.

[10] If students do not have a theoretical control/understanding over what the menu commands represent.

[11] For the analysis of other Cabri tools, such as dragging and measures, see (Arzarello *et al.*, 2002; Olivero, 2002; Olivero & Robutti, 2001).

example to avoid what happened to Debora and Giulia, when they had too many lines visible and could not identify which quadrilaterals they had to consider.

The previous discussion leads to broaden the perspective that considers dynamic geometry environments only as add-ons, i.e. as environments that provide students with resources that experts usually possess, and as such need to be abandoned at some stage in the learning process. This paper has shown that it is necessary to take into account the potentialities of this type of software, and more generally of new technologies, to generate new problems and perspectives with respect to paper and pencil, that affect *doing mathematics*. To conclude, I would like to paraphrase Godfrey's statement "Drawing is not to be abandoned at a definite epoch in the geometry course: practice and theory should advance hand in hand" (Godfrey, 1910), into "Cabri is not to be abandoned at a definite epoch in the geometry course: Cabri and theory should advance hand in hand".

REFERENCES

Arsac, G., Germain, G., & Mante, M. (1988). *Problème ouvert et situation - problème*. Academie de Lyon: IREM.

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in cabri environments. *Zentralblatt fur Didaktik der Mathematik, 34*(3), 66-72.

Fujita, T., & Jones, K. (2002). The bridge between practical and deductive geometry: Developing the 'geometrical eye'. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 384-391). Norwich, UK.

Godfrey, C. (1910). The board of education circular on the teaching of geometry. Mathematical association. *Mathematical Gazette, 5*, 195-200.

Hölzl, R. (2001). Using dynamic geometry software to add contrast to geometric situations - a case study. *International Journal of Computers for Mathematical Learning, 6*, 63-86.

Laborde, C. (2004). The hidden role of diagrams in pupils' construction of meaning in geometry. In J. Kilpatrick, C. Hoyles & O. Skovsmose (Eds.), *Meaning in mathematics education* (pp. 1-21). Dordrecht: Kluwer Academic Publishers.

Mariotti, M. A. (2000). Introduction to proof: The mediation of a dynamic software environment. *Educational Studies in Mathematics, 44*(1/2), 25-54.

Mariotti, M. A. (2002). Influence of technologies advances on students' maths learning. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh & D. Tirosh (Eds.), *Handbook of international research in mathematics education* (pp. 695-721): Lawrence Erlbaum Associates.

Olivero, F. (2002). *The proving process within a dynamic geometry environment*. Bristol: University of Bristol, ISBN 0-86292-535-5.

Olivero, F., & Robutti, O. (2001). Measuring in cabri as a bridge between perception and theory. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the international group for the psychology of mathematics education* (Vol. 4, pp. 9-16). Utrecht, The Netherlands.