Adaptive and Dynamic Hypertext Tutoring Systems Based on Knowledge Space Theory

Dietrich Albert, Cord Hockemeyer

Adaptive and Dynamic Hypertext Tutoring Systems Based on Knowledge Space Theory

Dietrich Albert Cord Hockemeyer
Department of Psychology, University of Graz
Universitätsplatz 2/III, A–8010 Graz, Austria
E–Mail: {Dietrich.Albert|Cord.Hockemeyer} @ kfunigraz.ac.at

Abstract

Knowledge space theory provides a formal model for representing students' knowledge and describing the structure of a domain of knowledge. A similar formal structure can be used to described the structure of hypertexts. The combination of knowledge space theory and the formal hypertext model leads to a framework for intelligent tutoring systems which provides individualized learning paths to a student. Using powerful procedures from relational database systems and from knowledge space theory, we get e. g. an efficient selection of appropriate teaching documents.

1 Introduction and previous results

Doignon and Falmagne [1] introduced the theory of knowledge spaces which provides a mean to formally describe the structure of a given domain of knowledge. We introduce the theoretical concepts in Section 1.1 below. The basic idea is the description of a student's knowledge by the set of problems (*items*) he or she is able to solve. The set of possible *knowledge states* is restricted by *prerequisite relationships* between the items.

A similar structure as knowledge spaces can be used to describe a hypertext: prerequisite relationships between different hypertext components can be specified by *prerequisite links* [2]. These prerequisite links build a structure which is comparable to a knowledge space. In section 1.2, we describe a formal model for hypertext structures.

In Section 2, we describe how the combination of the formal hypertext model and the theory of knowledge spaces leads to an individualized and efficient hypertext tutoring system.

1.1 Formal concepts of knowledge space theory

The formal concepts in knowledge space theory have been introduced in [1]. Some more recent research developments can be found e. g. in [3].

In knowledge space theory, a field of knowledge is specified by a finite set of items, i. e. problems or tasks a student may or may not be able to solve. Each student can be described by his/her *knowledge state*, i. e. the subset of items which this student masters. However, since there exist prerequisite relationships between the items, not all possible subsets of items are knowledge states. The set of all possible knowledge states is called a *knowledge space*. Such a knowledge space contains the empty set $\emptyset$ and the complete item set Q as elements, and, for any two knowledge states $K, K' \in \mathcal{K}$, their union $K \cup K'$ is also a member of $\mathcal{K}$.

We have already mentioned that the set of possible knowledge states is restricted by prerequisite relationships between the items. In knowledge space theory, these relationships are formalized by *surmise relations*, i. e. quasi–orders on a set Q of items. Such a surmise relation $\sqsubseteq$ may be interpreted as $q \sqsubseteq q'$ if and only if from a correct response to problem q' we can surmise a correct response to problem q. Instead, we could also say that mastering item q is a prerequisite for mastering item q'.

1.2 A mathematical model for the structure of hypertext

Albert, Held, and Hockemeyer [2] have suggested a mathematical model for the structure of a hypertext using the terminology of the Dexter hypertext reference model [4]. Within the Dexter model, a *hypertext* consists of a set of *components* and a set of *links* between these components. A component consists of a *base component* (unit of information) and some *source* and *destination anchors* which are located on the base component. Links are specified by *source anchors* and *destination anchors* and the components on which the anchors are located. In the following, we present the core of [2].

We assume the existance of three sets B, S, and D of base components, source anchors, and destination anchors, respectively. A *component* $c = (b, S_c, D_c) \in (B \times 2^S \times 2^D)$ is a triple constituted by a base component $b \in B$, a subset $S_c \subseteq S$ of source anchors, and a subset $D_c \subseteq D$ of destination anchors. A set $L \subseteq (C \times S) \times (C \times D)$ where each $l = ((c,s)\,,\,(c',d)) \in L$ fulfills the condition $s \in S_c$ and $d \in D_{c'}$ is called a *set of links.* A pair $H = (C, L)$ is called a *hypertext.* A link $l = (\,(c, s_c)\,,\,(c', d_{c'})\,)$ connects a source anchor s_c located on a component c with a destination anchor $d_{c'}$ on a component c'.

Based on this formalization of links, we introduce a binary relation $\vdash$ which describes the linkage between components. For a Hypertext $H = (C, L)$, we define the *link relation* $\vdash\subseteq C \times C$ on the set C of components. For any components $c, c' \in C$, we obtain $c \vdash c'$ if and only if there exists a link $l = (\,(c, s)\,,\,(c', d)\,) \in L$.

2 Combining hypertext structures and knowledge space theory

Knowledge space theory and the mathematical hypertext model can be combined to build a framework for intelligent hypertext tutoring systems. As a central point in the combination of knowledge spaces and hypertexts we use the concept of prerequisite relationships. These prerequisite relationships are specified using *prerequisite links* which is a special example of a *link type.*

2.1 Prerequisite links in dynamic hypertexts.

Albert, Held and Hockemeyer [2] introduce the concept of link types. Assume a hypertext $H = (C, L)$ as defined in Section 1.2 above. For an arbitrary subset $P \subseteq S$, we call a link $l = (\,(c, s)\,,\,(c', d)\,)$ a *link of type* P if and only if $s \in P$. We denote by $L^P = \{l = (\,(c, s)\,,\,(c', d)\,) \in L \mid s \in P\}$ the set of all links of type P. We define a binary relation $\vdash^P \subseteq \vdash \subseteq (C \times C)$. For any components $c, c' \in C$, we obtain $c \vdash^P c'$ if and only if there exists a link $l = (\,(c, s)\,,\,(c', d)\,) \in L^P$. This relation $\vdash^P$ is called *P–relation.*

In this paper, we make only use of the *prerequisite links.* A prerequisite link from a component c to a component c' (i. e. $c \vdash^P c'$) means that c' is a prerequisite for c, i. e. for understanding the contents (or solving the problem) of component c, the content of component c' must be known. Semantically, this prerequisite relation has a similar meaning to that of the surmise relation with the components corresponding to the items. While we use the surmise relation to select new problems during the student's knowledge assessment, we can use the prerequisite relation to select appropriate components to be offered to the student.

2.2 Individualized learning path.

The transfer of the concept of surmise relations from knowledge spaces to hypertexts enables us to use procedures developed within the frame of knowledge space theory. One of these concepts is the *learning path* [5].

Let $H = (C, L)$ be a hypertext and let $\vdash^P$ be a prerequisite relation on C as specified in Section 2 above. A sequence $X_0, X_1, \ldots, X_n$ of subsets of components is called a *learning path from X_0 to X_n* if and only if the following two conditions hold. (i) For any $i = 0, 1, \ldots, n$, for any $c \in X_i$, and for any $c' \in C$, if $c \vdash^P c'$ then $c' \in X_i$ and (ii) for any $i = 1, 2, \ldots, n$, there exists a $c \in C \setminus X_{i-1}$ such that $X_i = X_{i-1} \cup \{c\}$. A learning path from $\emptyset$ to C is simply called a *learning path*. Condition (i) ensures that the steps of the learning paths are states, i. e. subsets of components which conform to the prerequisite relation. Condition (ii) provides that the learning path describes a step–by–step learning.

Individualized learning paths are constrained by the educational objective and the student's current knowledge state. The former may be specified by the student or by some teacher while the latter can be determined with assessment procedures known from knowledge space theory [6, 3]. In general, there will exist several learning paths from the starting to the destination state [5].

During the tutoring process, the student's knowledge state can be updated at any step corresponding to the lessons learned. At any time, the student has access to those components which are the next step on some learning path from the current state to the destination state.

3 Discussion

In this paper, we suggest the combination of knowledge space theory and a mathematical hypertext model to build efficient and adaptive intelligent tutoring systems. The efficiency of such a system has two sources: First, knowledge space theory provides us with efficient procedures based on different representations (surmise relations, knowledge spaces and their bases, prerequisite relations) for the structure of a domain of knowledge. We can, for example, implement storage– and time–efficient procedures for the assessment of knowledge using a theorem of Birkhoff. Second, the formal description of hypertext structures using mathematical relations enables us to apply methods from relational database theory to these structures quite directly. This concerns not only individual access to document appropriate for the student's actual knowledge but also the construction of sub hypertexts due to educational objectives or to the student's prior knowledge.

We also want to mention some extended concepts from knowledge space theory. One idea is the generalization of the surmise relation to a *surmise system*, which reflects the fact that there may exist several ways to solve a problem. Another, recent extension is the distinction between observable behaviour (items or performance) on the one hand and underlying skills (or competencies) on the other hands. Such extensions have been suggested by Doignon, Düntsch and Gediga, and Korossy.

These extensions of the knowledge space theory can lead to an extension of the framework for intelligent hypertext tutoring systems.

References

[1] Jean-Paul Doignon and Jean-Claude Falmagne. Spaces for the assessment of knowledge. *International Journal of Man-Machine Studies*, 23:175–196, 1985.

[2] Dietrich Albert, Theo Held, and Cord Hockemeyer. A relational model for describing hypertext structures. 1997. Submitted for publication.

[3] Dietrich Albert, editor. *Knowledge Structures.* Springer Verlag, New York, 1994.

[4] Frank Halasz and Mayer Schwartz. The Dexter hypertext reference model. *Communications of the ACM*, 37(2):30–39, 1994.

[5] Jean-Claude Falmagne. Stochastic learning paths in a knowledge structure. *Journal of Mathematical Psychology*, 37:489–512, 1993.

[6] Cornelia E. Dowling, Cord Hockemeyer, and Andreas H. Ludwig. Adaptive assessment and training using the neighbourhood of knowledge states. In Claude Frasson, Gilles Gauthier, and Alan Lesgold, editors, *Intelligent Tutoring Systems*, volume 1086 of *Lecture Notes in Computer Science*, pages 578–586, Berlin, 1996. Springer Verlag.

Acknowledgments

This work was supported by the grant ERBCHBICT941599 in the program on human capital and mobility (HCM) of the Commission of the European Communities. The authors are grateful to Theo Held for his valuable suggestions.