APLUSIX, A learning environment for algebra, actual use and benefits

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Summary

After a presentation of the APLUSIX system, a learning environment for formal algebra, we describe the use of this system in a French class of grade 10, in particular a sequence devoted to factorisations and equations, and an analysis of the students' work during this sequence. This study exhibits the influence of the tools provided by the software on the construction of competences in algebra. We also report the opinion of a teacher who used the APLUSIX system during one year with this class.

1 Introduction

Several kinds of difficulties in the learning of algebra have been revealed by researches in mathematics education: the transition of the reasoning mechanism from arithmetic to algebra (Kieran, 1992); the status of letters and the notion of variable in algebraic expressions (Kieran, 1991), the evolution from procedural to structural conceptions (Sfard, 1991). The elaboration of relevant learning situations to surpass these difficulties is a fundamental goal in mathematics education. The environments allowing to build learning situations for algebra (Sutherland & Rojano, 1993) generally provide little appropriate feedback.

This paper describes a learning environment for formal algebra, named APLUSIX, and presents a study on its capacity to be a *milieu* (Brousseau, 1997), for the learning of algebra. This study is mainly based on the use of the system in a French class of grade 10 during the entire school year 2002-2003. The main part which is developed in the paper concerns a teaching sequence devoted to factorisations and equations. The analysis of the students' behaviour exhibits the features of the software as a learning *milieu* and shows its influence on the construction of competences in algebra.

2 Description of APLUSIX

APLUSIX (Bouhineau et al., 2002) is a learning environment for formal algebra. This system includes an advanced editor of algebraic expressions that displays the expressions in the usual form and allows the modification of the expressions in this form. This editor is based on the structure of algebraic expressions (Kieran, 1991) for the upper functions (selection, cut, copy, paste, drag and drop), e.g., only algebraic sub-expressions can be selected. As a consequence, the students make their own calculations, as they do with paper and pencil. This is very different from the other learning environments for formal algebra that require the use of a command to perform each action (Beeson, 1996), e.g., to commute 4 and x in 4+x, one has to apply a command containing the identity a+b=b+a, instead of writing x+4 or performing a drag and drop of x. Besides this heavy interaction mode, most of these systems do all the calculations. In these command-based environments, the student cannot make an error and cannot learn from the correction of his/her errors.

With APLUSIX, the students freely develop calculation steps that are rectangles containing the expressions. These steps are verified by the system which calculates the equivalence of the expressions and shows the result, see figure 1. APLUSIX also provides information on the state of the current step, see figure 1.

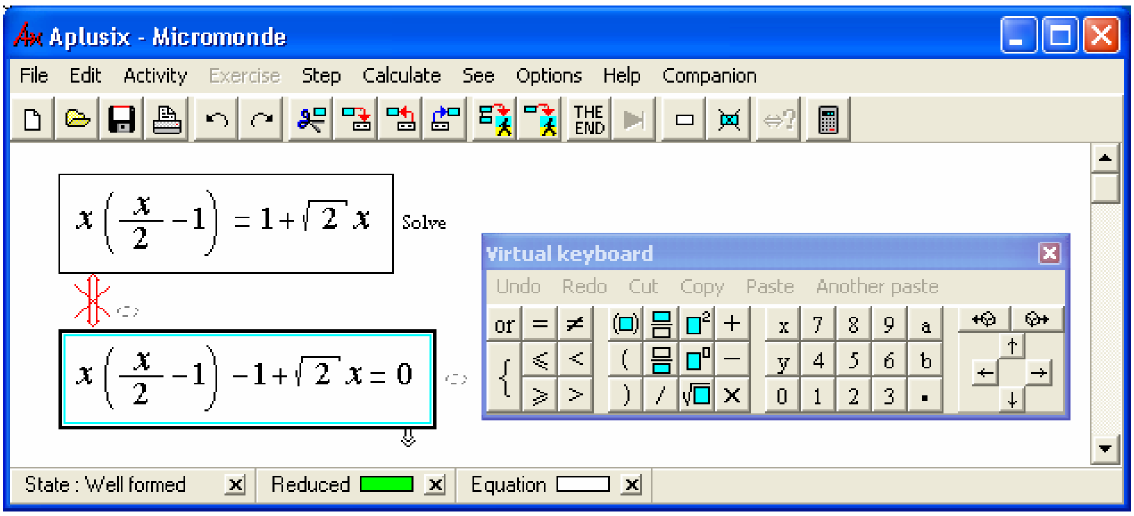

Figure 1. The student moved the right hand side of the equation to the left and changed only one sign. A vertical, red and crossed equivalence sign between the two boxes indicates an error. The status bar, at the bottom of the window, indicates that the expression of the current step is reduced and is not a significant step in the equation solving process.

At the end of the exercise, APLUSIX verifies that the last step contains a solved form, checks that the path going to this step is made of correct steps and provides a feedback like *the exercise is solved*, *this is not a solved form*, *you still have to reduce*, *the path going to the current step contains errors*. When the exercise is not solved, the student can continue the solving process or leave.

Nevertheless, APLUSIX has also a few commands: *calculate* for numerical expressions, *expand-and-reduce* and *factor* for polynomial expressions, *solve* for equations and inequalities. When they are available, they perform an action on the selected expression as Computer Algebra Systems do.

APLUSIX provides tools to the teacher: (1) to customize the system (with/without the verification of the calculation, with/without the status bar, with a field for each command, e.g., *solve* for equations can be disabled, enabled for first degree equations or enabled for second degree equations); (2) an editor allowing to build lists of exercises; (3) a component allowing to replay the student's actions later.

3 A *milieu* for the learning of algebra

According to the constructivist approach of (Brousseau, 1997), "The student learns by adapting herself to a *milieu* which generates contradictions, difficulties and disequilibria, rather as human society does. This knowledge, the result of the student's adaptation, manifests itself by new responses which provide evidence of learning."

This *milieu* must be organised by the teacher through relevant choices of: (1) types of action of the student; (2) type of feedback of the system; (3) exercises. A computer system with relevant exercises can be a milieu (Laborde, 2000). This is the case of APLUSIX. The actions are the functions of the editor (input, delete, copy, paste…) and the commands for having automatic calculations. There are three kinds of feedback: (1) the checking of the equivalence between two steps; (2) the indications of the status bar; (3) the messages displayed at some moment, e.g., at the end of the exercise.

The main feedback is the checking of the equivalence between two expressions. This equivalence is based on the fundamental semantics of algebra, usually called denotation (Arzarello et al., 2001; Nicaud et al., 2001): polynomial expressions are equivalents when they correspond to the same polynomial; equations are equivalent when they have the same set of solutions, etc. Concerning the equivalence, the teacher can choose between three modality of the feedback: a permanent feedback (displayed at each modification); a feedback on demand of the student (by a click on a button); no feedback. In this last case, a simple line is drawn between the boxes and the system mainly works as an editor based on the structure of the expressions and indicating ill-formed expressions.

The checking of the equivalence is an important feedback. However, the student also needs feedback that allows to give sense to expressions according to his/her goals (Sfard, 1991; Harper, 1987). The status bar of APLUSIX shows part of this sense: degree of factorisation, of expansion, of reduction; progression in the equation solving process, etc.

As said above, the teacher can customize the system (by choosing the values of 30 parameters). This allows the teacher to get different kind of *milieu* and to choose the most relevant for his/her teaching goal.

4 A sequence on equation solving at grade 10

In this section, we present a use of the APLUSIX system in a French class of grade 10 having 33 students. We worked with this class during the entire school year 2002-2003 on the algebraic part of the curriculum. In September 2002, before any teaching of algebra, we gave a pre-test to the students. This test was made with the APLUSIX environment and contained exercises of the different algebraic topics of grade 9. The analysis of the students' behaviour revealed remaining difficulties for applying the knowledge learned in grade 9. As a consequence, we prepared a teaching sequence on two topics: factorisation and equation solving. This sequence was a list of exercises that the students had to solve in the APLUSIX environment, using the feedback of the system.

4.1 Equation in the French class 'seconde' (grade 10)

From grade 8 to grade 10 in France, the curriculum contains equations that can be classified in the following types, where x is the unknown, a, b, c, and d are numbers, the coefficients of x being different from 0:

T_0: $ax+b=0$ T_1: $ax+b=cx+d$ T_2: $(ax+b)\times(cx+d)=0$

T_3: $(ax+b)\times(cx+d)=(ex+f)\times(gx+h)$ with the three following sub-classes:

T_{3a}: After expansion, terms in x^2 disappear, factorisation does not apply

T_{3b}: After expansion, terms in x^2 disappear but factorisation applies too

T_{3c}: After expansion, terms in x^2 do not disappear, factorisation is necessary

T_4: Other types, e.g., $x^2=a$, that can be transformed into a previously described type.

Taking into account this classification, we set up a learning sequence aiming at showing the utility of the factorisation for the equation solving process. The work of the students on factorisations was limited to the factorisations that help to solve equations.

4.2 Methodology

From the results of our analysis of the problems of types T_1, T_2 and T_3 of the pre-test, we prepared a teaching sequence organized in three phases. During these phases, the students worked individually and exclusively with APLUSIX.

Phase 1: Acquisition of knowledge

The verification of the calculations was on demand and the indicators were shown to the student. In order to exhibit the use of factorisations for problems of type T_{3c}, we set up three steps:

- Step 1 (1 hour): Resolution of linear equations and second degree equations without factorisation. (problems of types T_1, T_2 et T_{3a} and T_{3b}).

- Step 2 (1 hour): Resolution of second degree equations with factorisation (problems of type T_{3c}). After steps 1 and 2, the teacher presented a synthesis on the procedures for solving equations.

- Step 3 (1 hour): Training with equations of type T_1, T_2 and T_3. For problems of type T_3, the students had to decide which procedure had to be used.

Phase 2: Post-test. A post-test has been organized in order to measure the student's evolution in comparison with the pre-test, on equations of type T_1, T_2 and T_{3c}. During this phase, the verification of the calculations was disabled and the indicators were hidden.

Phase 3: Individualized help. The post-test showed a group of five students in great difficulties. So we set up an individualized help for those who were volunteers (four over five). This work was done outside of the class, at home, or in a self-service computer room.

4.3 Results

Table 1 shows the evolution of the score of the students for the problems of types T_1, T_2 and T_{3c}. The evolution depends on the type of the problem, and also, on subcategories of problems. For example, about T_1, the success for exercises with integer coefficients was 100% in the post-test whereas it was only 30% for exercises with non integer coefficients.

As students only worked with APLUSIX during this period, we make the hypothesis that the evolution of the results is due to the *milieu* provided by APLUSIX and to the choice of the situations.

Type of problems	Pre-test	Post-test
T_1	46%	74%
T_2	3%	63%
T_{3c}	27%	71%
Total	18%	68%

Table 1: evolution between the pre-test and the post-test (percentages of well solved exercises).

4.4 APLUSIX as an environment for doing experiments

During phase 1, the verification of the calculations was available. The students had to decide to ask for the display of the equivalence at some moment, and to use the resulting information. This introduces APLUSIX as an environment for doing experiments. While performing actions and observing feedback, students developed their abilities, their controls, their strategies, and were able to correct their errors (see figure 2).

$(x+2)\ (x-3) = (x+2)\ (x-4)$ Solve $(x-3) = (x-4)$ 1) Mary duplicates the given equation. Then, she selects and deletes (x+2) on each side.	$(x+2)\ (x-3) = (x+2)\ (x-4)$ Solve $\not\Updownarrow$ $(x-3) = (x-4)$ 2) She clicks on the verify button. She gets a red and crossed arrow indicating a non equivalence.
$(x+2)\ (x-3) = (x+2)\ (x-4)$ Solve $x^2-x+2x-5=x^2-4x+2x-8$ 3) Mary deletes the equation of the second step and inputs an expanded form of the given equation.	$(x+2)\ (x-3) = (x+2)\ (x-4)$ Solve $\not\Updownarrow$ $x^2-x+2x-5=x^2-4x+2x-8$ 4) She clicks on the verify button and gets again a non equivalence answer.
$(x+2)\ (x-3) = (x+2)\ (x-4)$ Solve $x^2-3x+2x-6=x^2-4x+2x-8$ 5) Mary deletes 5 and insert 6 at this place. Then she inserts a 3 for changing –x in –3x.	$(x+2)\ (x-3) = (x+2)\ (x-4)$ Solve $\Updownarrow$ $x^2-3x+2x-6=x^2-4x+2x-8$ 6) She clicks on the verify button and gets an equivalence answer.

Figure 2. Beginning of the resolution by a real student Mary of the exercise "Solve the equation $(x+2)(x-3)=(x+2)(x-4)$ *". The equivalence between two steps is indicated to Mary when she clicks on the* verify *button*

During step 2 of phase 1, the students tried to expand the equations of type T_{3c}. They reached an equation of the form $ax^2+bx+c=0$ that they were not able to solve. At this moment, the teacher made an intervention to explain the factorisation strategy. Then the students applied this strategy to equations having apparent factors. After a while, some students used again the expand strategy for second degree equations. Two hypotheses can explain this behaviour: (1) the factorisation method was not yet well established; (2), the common factor was not apparent enough, e.g. $(2x-4)(x+1)-(x-2)(x+3)=0$. After they got the $ax^2+bx+c=0$ form, most of these students backtracked to a previous step and used the factorisation method.

Globally, the teacher observed that the students changed their answers more easily than in the paper and pencil environment, and did not hesitate to test new strategies.

5 The teacher's point of view after one year of use of APLUSIX

During the rest of the year, the teacher was invited to use APLUSIX every time he thought it was relevant. We observed that he used APLUSIX every time he worked on algebra, especially for inequalities and systems of equations, alternatively with the paper-pencil environment. Overall, during the school year, the students used APLUSIX during 12 hours with one student per computer.

Regularly, we discussed with the teacher about the use of APLUSIX in his class. We present, below, the main points evoked during these discussions:

- The discovery of the system does not present any difficulty and is very quick.
- Students in difficulty have shown motivation to solve exercises, on their own, out of the lesson. They chose exercises in their text book and solved them with APLUSIX. This can be explained by the fact that the students do not usually have enough control to validate their answer and that the system provides such validation.
- The students needed less help from the teacher when they were working with APLUSIX (this point was emphasised by the teacher). They gained autonomy for the solving of problems. This confirms the aspect of APLUSIX as an environment for exploring and experimenting.
- The teacher observed better results in comparison to those of the previous years, and students were more rigorous on syntactic aspects in the paper and pencil environment. This appeared particularly for systems of equations. During the use of APLUSIX, the students were not allowed to work on an equation then on another; they were obliged to make steps containing equivalent systems. Going back to the paper-pencil environment, most of them continued to work with equivalent systems.

6 Conclusion

We have shown that the use of APLUSIX can be easily integrated in the teaching of a secondary class, alternatively with the paper-pencil environment. After the first sequence devoted to factorisations and equations which was prepared by researchers, the teacher had no difficulty to use the software for the other parts of the algebraic curriculum. His opinion is very positive at the level of the usability, of the utility (the learning), of the autonomy and of the individualisation.

The results confirm our didactical analysis concerning the dimension of the availability of the verification of the equivalence. APLUSIX can be viewed as a *milieu* for validation, in the sense given by Brousseau (1997), as the student can know if his/her answer is correct or not, without the intervention of the teacher. This can reduce the effect of the didactical contract where students try to guess the result expected by the teacher when he is asked for validation. We also consider APLUSIX as an environment for experimentation as the interaction between the subject and the *milieu* allows the exploration and the evolution of the strategies.

Theses results are confirmed by other experiments we had during two years, with several classes of grades 9, 10 and 11 (Nicaud et al. 2003). Several of these experiments consisted in a use of APLUSIX with a little help of the teacher during this use and without any teaching of algebra in the regular class. They all showed an improvement of the competences of the students. The scores were measured by a pre-test and a post-test. They often were multiplied by two for similar exercises after 2 or 3 hours of use of the system.

APLUSIX is currently a prototype that can be downloaded from: http://aplusix.imag.fr for testing and using it. It runs in French, English, Portuguese, Italian, Japanese, and can be easily translated in other languages. It will become a commercial product in 2004.

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