# Towards a narrative-oriented framework for designing mathematical learning

Yishay Mor, Richard Noss

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> *Actually, it is half the art of storytelling to keep a story free from explanation as one reproduces it. [...] The most extraordinary things, marvelous things, are related with the greatest accuracy, but the psychological connection of the events is not forced on the reader. It is left up to him to interpret things the way he understands them, and thus the narrative achieves an amplitude that information lacks.* Walter Benjamin, The storyteller

## Introduction

This paper proposes a narrative-oriented approach to the design of educational activities, as well as a CSCL system to support them, in the context of learning mathematics.

Both Mathematics and interface design seem unrelated to narrative. Mathematical language, as we know it, is devoid of time and person. Computer interfaces are static and non-linear. Yet, as Bruner (1986; 1990) and others show, narrative is a powerful cognitive and epistemological tool. The questions we wish to explore are –

- If, and how, can mathematical meaning be expressed in narrative forms – without compromising rigour?
- What are the narrative aspects of user interface? How can interface design be guided by notions of narrative?
- How can we harness the power of narrative in teaching mathematics, in a CSCL environment?

We begin by giving a brief account of the use of narrative in educational theory. We will describe the environment and tools used by the WebLabs project, and report on one of our experiments. We will then describe our narrative-oriented framework, by using it to analyze both the environment and the experiment described.

## Narrative approaches in education

A narrative, in a nutshell, is an account of *something* happening to *someone* in some *circumstances*. Jerome Bruner (1986, 1990) identified narrative as a fundamental vehicle for transmission of meaning. For

[1] We acknowledge the support of the European Union, Grant # IST-2001-32200, directed by Prof. Richard Noss and Prof. Celia Hoyles. http://www.weblabs.eu.com

example, research tracing caregivers' conversation with children found it rich in narratives. These serve the purpose of bringing children into the parents' culture. They are not merely descriptions of *what* happened, they provide an implicit or sometimes explicit explanation of *why* it happened.

Lakoff & Núñez (2000) describe a linguistic structure called *aspect schema*: "the general structuring of events. Everything that we perceive or think of as an action or event is conceptualized as having that structure" (35). This fundamental structure is derived from the general form of neural motor-control programs. The components of this schema can be grouped into three sections: preparatory or pre-action, flow of events, and completion. These correspond to the exposition, plot and closure of a narrative. Thus, the structure of narrative is fundamental because it is the natural verbalization of an underlying conceptual framework, derived from a basic mental capacity.

Given the strong cultural (perhaps even neurological) grounding of narrative it seems we should strive to embed narrative structure in design of systems or activities which are aimed at meaning-making. Yet narrative approaches to CSCL are focused primarily on designing systems that support narrative-based learning (Mott, 1999; Decortis, 2002, 2004), i.e. systems that support participants learning *to* tell stories, rather than learning *by* telling stories. Nehaniv (1999) argues for a wider view, claiming that design that does not acknowledge the "narrative grounding" of humans will appear to its users as bizarre, unintelligent and unintelligible.

> … it is desirable to take into account that humans are temporally grounded, narratively intelligent beings. Their evolutionary heritage leads them to expect that the actions of others are embedded in a context of past history and future events."

The design of the *WebReports* system described below acknowledges narrative structure by providing a context for each document, supporting a strong sense of authorship, and exposing the temporal structure of the content.

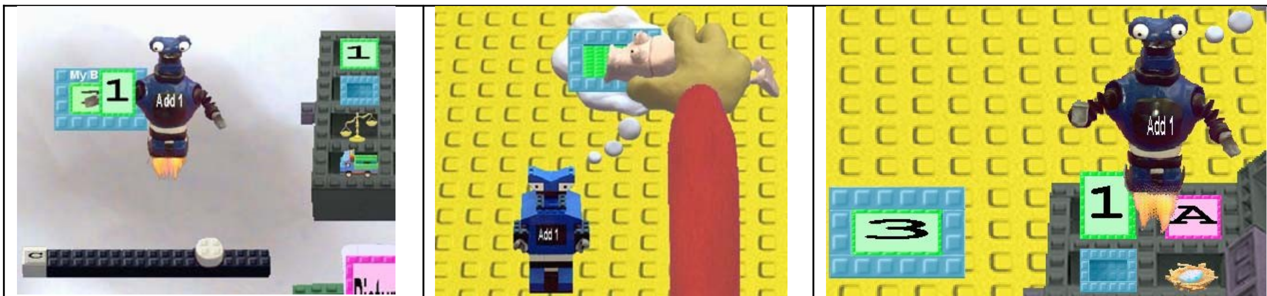
Leone Burton (1999) argues that the aim of providing the learner with tools for coping with unfamiliar knowledge points to a need to facilitate learners' authoring of their accounts of how they came to know mathematics. These narratives are personal, i.e. imaginative, as they are general and paradigmatic.

We see even wider implications for teaching mathematics: just as a user interface stripped of time, person and context appears as "bizarre, unintelligent and unintelligible", so does mathematical knowledge. We claim that in order to make such knowledge accessible, it needs to be situated in a context, maintain a sense of authorship, and acknowledge temporal structure. We also claim that this can be done without making the content any less mathematical. These assertions guide the design of the *Guess my Robot* activity, which is a centrepiece of this paper.

## *WebLabs*

*WebLabs* is a 3 year EU-funded educational research project oriented towards finding new ways of representing and expressing knowledge in communities of young learners. Our work focuses on the iterative design of exploratory activities in domains such as numeric sequences, probabilistic thinking, and fundamental kinematics. *WebLabs* utilizes two main media for its activities: *ToonTalk* (a programming environment) and *WebReports* (a web-based collaboration system). Using this infrastructure we design tasks and *transparent modules* – programming tools which students use to accomplish their tasks, but can also take apart, understand, and restructure.

We see software programming as playing a key role in individual and group learning. Children explore and test their conceptions through programming. Furthermore, by sharing programmed models, they communicate ideas in an intuitive yet univocal form. We are programming with *ToonTalk*[2] (Kahn, 1996; 1999) a language used in the past with younger children to construct video games (Hoyles, Noss & Adamson, 2002). In *ToonTalk* programs take the form of animated cartoon robots. Programming is done by training these robots: leading them through the task they are meant to perform. After training, programs are generalised by "erasing" superfluous detail from robots' "minds".




| | | |
|---|---|---|
| Train the robot to take a number 1 from the toolbox and drop it on the input, to increment it. | Generalise the program by erasing the value of the input from the robots memory. | Give the robot its input box. The robot will continuously repeat the actions it has been taught. |

Figure 1: Training a robot to count

Figure 1 shows three snapshots of what it means to write a program (train a robot) to count through the natural numbers. In fact, we only have to train the robot to "add 1" to a number and then *generalise* it to any number. The robot iterates the actions it was trained to do, for as long as the conditions it expects hold true.

The individual and collaborative facets of learning are intertwined at all stages of our activities. The *WebReports*[3] system (Figure 2) was set up to support both. A *web report* is a document which is composed and displayed online, through which a learner can share experiences, questions and ideas derived from her activities. The uniqueness of our system is that it allows the author to share her ideas not just as text, but as animated ToonTalk models. This last point is crucial: rather than simply discussing what each other *thinks*,

[2] *ToonTalk* is a commercial product. Free trial and Beta versions are available from http://www.*ToonTalk*.com

[3] http://www.weblabs.org.uk/wlplone/ . Our system is based on the open-source Plone framework (http://plone.org).

students can share what they have *built* and *rebuild* each others' attempts to model any given task or object. Reports are edited using a visual editor. On top of standard text formatting features, this editor allows users to easily embed media objects in their reports. Most importantly, students can grab any object in their *ToonTalk* environment, and copy it instantaneously into their report. The object is shown as an image, but it is also a hyperlink. When clicked by a reader the object will open in her *ToonTalk* environment – which could be in another classroom or another country.

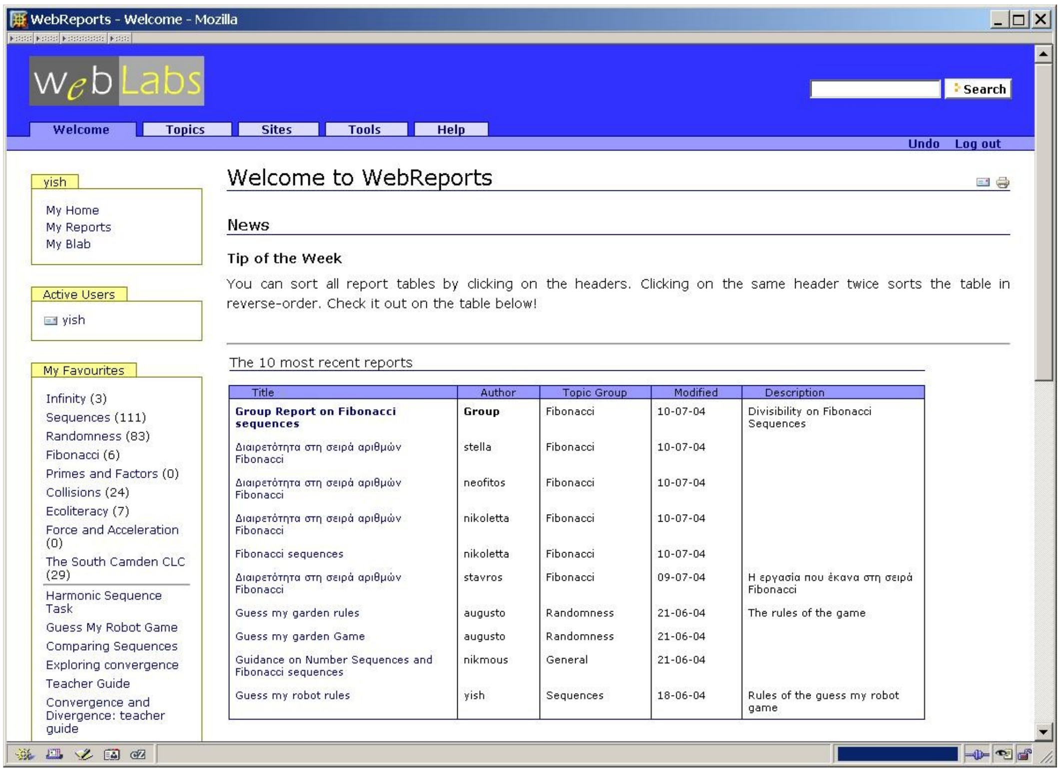



Figure 2: WebReports front page

Reports are catalogued along three axes: topic, site and function. The first categorizes reports by their subject content (e.g. Infinity, Sequences, 1D collisions). The second lists the reports by the real-world team of the author (school, class or club). The function heading presents content by the way it was conceived to be used (programming component, personal report, tutorial, etc.).

A comprehensive description of the system, including the design and experimentation process by which it was created, will be the subject of a separate paper. We restrict ourselves here to two features which play a critical role in the story we will tell below.

The first is the capacity to comment on others' reports. Each report page ends with a comment section. Any reader can add comments by entering a subject line and clicking the "go" button (Figure 3). The system provides a selection of standard subject lines, but also the option of typing a non-standard subject. The types of comments were inspired by CSILE (Scardamalia et al, 1989) and adjusted to our domain. The same editor used for reports is also available for writing comments, including the option of embedding

ToonTalk objects. Comments automatically note their creation date and author. They can be threaded hierarchically. The report author can delete unwanted comments, but not edit them.

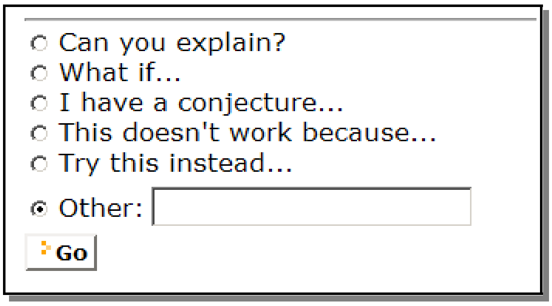



Figure 3: Comment creation form

The second feature we wish to detail is the *report template* mechanism. Through our observations of learners' interaction with previous prototypes of the system, we have identified the importance of scaffolding their authoring process. We choose to support this scaffolding by providing *report templates*. These are prefabricated skeletons of reports, which include headers, questions and occasionally ToonTalk tools. Like the commenting tool, templates are a "*soft scaffold*"; they suggest a structure but do not impose it. A user can start a report from a template, and once she does – the report is hers to edit at will. To use a template, the user selects it in the 'report generation' form, or navigates to the template and clicks a button on it (see Figure 4).

At first, we introduced these templates as a way of structuring students reflections on an activity, e.g. by providing headings such as "*What I did*" and "*What I learnt*". With time we discovered more and more uses for this facility, notably a method of creating "active worksheets": reports which include tools and instructions for an activity. Students replace the instruction text with their reflections and products as they go along.

## The Guess my Robot game

One of the activities we designed was the *Guess my Robot* (GmR) game. This game is a pivotal activity in our explorations of number sequences.  Most students enter it with very little formal knowledge of sequences, and minimal ToonTalk experience. After GmR they move on to more advanced topics, such as the Fibonacci sequence, convergence and divergence, and cryptography. See Mor et al (2004) for a discussion of the mathematical-educational context of this game.

In this game, *proposers* train a robot to generate a numerical sequence, and publish its first few terms as a ToonTalk "box" in a WebReport, using a special purpose template. *Responders* build a robot that will produce this sequence, and thus show that they have worked out the underlying rule. As one girl said: "*So, like, the robot is my proof that I got it?*"

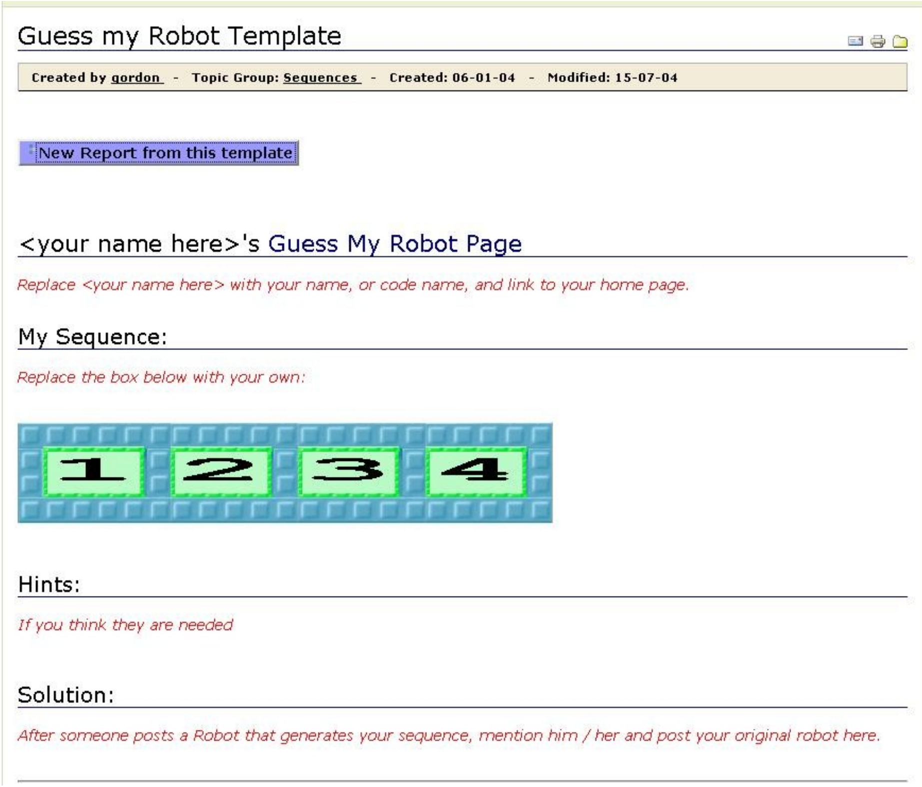



**Figure 4: GmR template**

We first experimented with this activity in 2002/3 (Mor and Sendova, 2003). In 2003/4 we expanded the experiment, with a collaboration system informed by the needs of the game. This iteration included 33 students from 6 sites (each in a different European country). Overall, 45 challenges and 33 responses were posted. However, only 17 of the challenges received a response at all. For the remainder of the paper, we will focus on two stories which arose from this game. The first is the story of Rita[4], the sequence she posed, and the interactions that it sparked off. The second is the story of Joe999 and his enculturation into the game practice.

## Rita's challenge

Rita is a 14-year-old girl from Lisbon, who has been participating in *WebLabs* since February 2003. She likes maths, but had not yet learnt much about sequences in school: this topic is not highly developed in the Portuguese curriculum. In fact, most of her experience in this topic comes from her involvement with *WebLabs*.

[4] We use the aliases, or "handles" children chose for themselves in the web reports system. With the system's access restrictions in mind, we can use this as an anonymized identifiers.

Rita found the 'guess my robot' activity, and decided to pose her own challenge:

**Rita's *Guess My Robot Page***

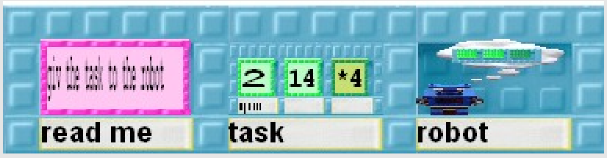
My Sequence:

I created this sequences of numbers. My five firsts terms of the sequence are:


A few days after she posted it, the Sofia *WebLabs* group held a session, and some of the students tried solving Rita's challenge. Nasko, a 12 year-old boy, posted his response:

**Hi Rita,**

Your sequence is very nice, but I have managed to solve it. See the solution below.




Best regards NASKO!!!

He had built a robot that produced Rita's five terms, but also realised that the same robot could be used to generate other sequences by changing its initial inputs. So, he posed a two-part challenge for Rita: he posted a different sequence (*9.5, 14, 16.25, 17.375, 17.9375…*) and asked:

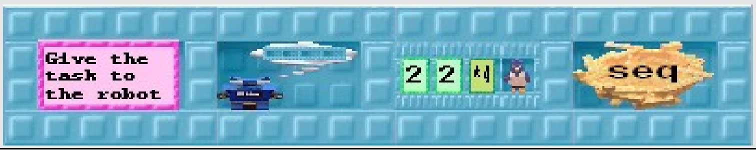
> *Here is also a sequence generated by the same robot. Two questions:*
>
> *1. What was the input of my robot?*
>
> *2. Can your robot generate it?*

A few days later Rita came to her next session. She was very excited to find comments on her page – and from children on the other side of Europe! She immediately clicked on the *ToonTalk* robots in the responses, and watched them step through the process of rule-generation. She was totally surprised: Nasko and Ivan had solved her challenge, but their robots seemed completely different from hers (and one from the other)!

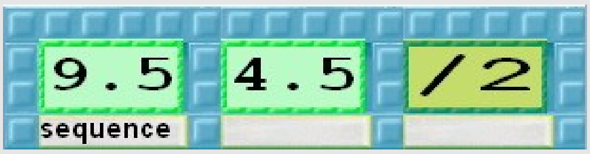
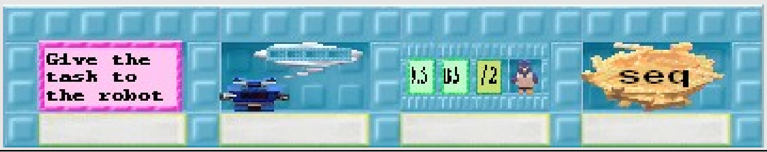
Rita posts a response to Nasko's comment:

Congratulations, you found a solution for my sequence!

But you used a different procedure of mine. I made thus:



And adds –

> I have a conjecture...
>
> About your questions:
>
> For your new sequence I thik  that is the imput for your robot:
>
> 
> 
>
> For my robot I make this:
>
> 
> 

After this interaction, the researchers in London intervened by posting several comments, with questions such as:

- Can you explain how you worked out the solution?
- How can you be sure that the robots will produce the same sequences forever?

Rita engaged in a high-level mathematical discussion with us, the details of which are reported in Mor et al (2004).

About a month later, the Cypriot group joined the game, and a group of students there came across Rita's challenge.

They posted a different robot, but packaged it using Rita's programming style. They added two comments to their own response, one listing the team members who contributed to the solution, the other telling the story of how they solved the challenge.

As before, Rita congratulates the Cypriots on their solution, provides her robot as reference, and concludes:

> *I can prove that my sequence and your sequence are equal with the process of algebraic representation used by Sofia group.*
>
> *Rita's sequence:*
>
> *A1 = 2*
>
> *An+1 = ( An + 2 ) x 4, but if I using the distributive property of the multiplication relatively to the addition I can write that:*
>
> *A1 = 2*
>
> *An+1 =  An x 4 + 8*

*that is the algebraic representation of the Chyprus's* [sic.] *sequence. then I can prove that two sequences are equal.*

## Narrative-oriented analysis of Rita's challenge

The case of Rita's challenge, and the guess my robot game in general, is one of the more successful stories of the WebLabs project. We claim that to a large extent, this success stems from the narrative features of the system and activity design. Other possible factors are suggested in Matos et al (2004). Here we look at the components of narrative mentioned above (genre, context, plot, voice), and reflect on the role they play.

# Genre

Bruner (1986) distinguishes between paradigmatic and imaginative modes of narrative:

> …the structure of a well-formed logical argument differs radically from that of a well-wrought story. Each, perhaps, is a specialization or transformation of a simple exposition, by which statements of fact are converted into statements implying causality. But the types of causality implied in the two modes are palpably different. The term *then* functions differently in the logical proposition "if x, then y" and in the narrative *recit* 'The king died, and then the queen died'

Paradigmatic narrative is the embodiment of mathematical-logical-scientific reasoning. It works top down, seeks generality and demands consistency. Imaginative narrative is the form of a good story. It is a bottom up discipline, seeks specificity and demands coherence.

In reality, mathematical ideas are often conveyed in an imaginative manner, and literary works may include paradigmatic fragments. Ivan, who is also a member of the Sofia group, could not participate in that particular *WebLabs* session. Still, that did not stop him from trying his hand at Rita's challenge, and posting his own response:

> *Today I couldn't attend my TT class because my shift is still in the morning and the rest of Sofia group go to school in the afternoon. So I decided to find a nice sequence to solve! And here was Rita's sequence. I used only two halls in the box. I'm curious to see other solutions.*

Most of the text is purely imaginative – Ivan gives a journal-style account of his day, referring to facts about his daily routine. His entry into the game is reported in a very affective tone, as if he decided to take a walk in the garden. Yet, one phrase refers to the structural details of his solution: "I used only two halls [sic. – holes, y.m.] in the box".

Some of the more interesting examples are those where genres are mixed. When asked by the London researchers how she solved Nasko's challenge, Rita answers with an imaginative-style account of events. Yet the implicit content – the moral – of her story is an algorithm:

> *In the Nasko's task to my sequence he used 2 like first term, 14 is the number that he used for add*

*a 2, and to get the second term (16), and for to multiply 4 .Then, I think to the Nasko's sequence the first number of the task it has that to be  9.5 because is the first term of him sequence, the second number of the task it has a number that he add to 9.5 for to get 14 (second term), this number is 4.5.*

*In my sequence he use the x4 for to get the third term (16 + 14 x 4), then in him sequence I think this: 14 + 4,5 "I don't know wath" it has that to be 16.25 or 4,5 "I don't know wath" it has that to be 2.25. But 2.25 is half of 4.5, then in third hole of the task I need to put /2.*

*After that I tested this task in Nasko's robot and it works.*

As a mathematical argument, this text is both superfluous in detail and inexplicit in specifying the big idea. Only by reading it as a narrative can we gain a window on Rita's intentions in writing it.

When designing a system which aims to support individual and group construction of knowledge, we need to acknowledge that authors will express themselves in multiple genres, and shift between them as appropriate to their goals or cognitive styles. We need to enable diversity, and at the same time provide cues which will help readers identify the genre authors have chosen. Keeping this balance was a central consideration in the design of the report templates and the comments tool, leading to our notion of soft scaffolding, mentioned above.

## Context

A narrative is always contextualized. It habitually begins with an exposition, which lays out the context: time, location, props and characters. Such an exposition is not limited to imaginative narrative. It also appears in scientific texts (Bruner, 1986).

Looking at Rita's game page, we find two levels of contextualization; automated and personalized. The first level of context is provided by the system in the course of a user's actions. Each report has a header (Figure 5), which lists the creation and last modification dates, and links to its author's home page and the topic groups she selected. The author's home page links to her site. The report header (and the information linked from it) functions as a structured exposition, contextualizing the report for readers.


Figure 5: Report header

Templates provide further context, such as familiar headings and structure, or activity specific hyperlinks. Combining our previous knowledge with the context provided by the header and template, we can read a lot from a report even when it is in a language we do not know – as demonstrated in Figure 5.

Looking closer at the text of Guess my Robot pages, we see that students choose to provide some context, typically in the form of an exposition, even in very short comments.

The Cypriot team decided to explain how they reached their solution:

> *We copied Rita's numbers in Excel, to be easier to find relations between the numbers and especially the differences.*
>
> *2. We found the differences between the numbers on that sequence.*
>
> *3. We noticed that differences between numbers could be calculated if we multiply every one difference by 4.*
>
> *4. So, we decided that we could work with formula 4* number.*
>
> *5. To get Rita's sequence, we had to add 8 to the previous formula. The final formula is 4*number+8*

Implicitly the students are describing an algorithm for solving such problems. This algorithm is the moral of the story about how they solved the challenge. The context of using Excel to find the differences and spotting the pattern have no mathematical significance. Yet they are what make the implicit mathematical and computational ideas approachable to any student who is familiar with the tool.

## Voice

The term voice relates to the presence of the speaker. Even in allegedly impartial arenas, such as scientific or legal writing, we attribute great significance to the voice of a document's author. When approaching a scientific paper, one relies heavily on knowledge of the author: her past publications, close collaborators, institution, etc. Likewise, when writing a paper, one is advised to imagine the readers and engage in a dialogue with them. Familiarity with the writer's personal style makes her writing much easier to understand and to learn from. Acknowledging voice enables us to leverage our narrative intelligence in reading, and writing, formal arguments. A clear sense of authorship promotes responsibility for the text. As it is enhanced, the text becomes part of the speaker's public image. On an affective level, we pay closer attention to the words of writers we know.

As we noted above, a WebReport's exposition links to its author. To be precise, it links to the authors' home page. From this page, the reader can see which site and topic groups the author is affiliated with, read other reports by her, and access her *blab* (a feature we will comment on shortly). The home page also leads to a table of all reports written by the author – even those which have not been publicly announced. Note that as they could not post a response as a group, the Cypriot team found it important to comment on their own posts and list their names.

In Matos et al (2004) we argue that the personal style of entries (challenges and responses) has a notable effect on interaction and engagement. Proposed challenges are identified with their authors. Responses are presented as personal comments. Students are not playing with the system, but with peers. If these peers are rude or arrogant, they will disengage. If they are friendly, the game will become collaborative rather than competitive. For example, participants always open their challenge or response with a highly personal exposition, such as: "*Hi Rita, your sequence is very nice, but I have managed to solve it.*"

## Plot

A well-formed narrative must maintain coherence of *temporality* and *causality* (Gergen, 1998). Temporality refers to the chronological ordering of events. In the light of narrative intelligence theory (Sengers & Mateas, 1999), it is clear that maintaining the temporal structure is crucial to the readers' ability to comprehend a story. The identification of temporal affinity of events also plays a strong role in learners' inferences of causality, which are an important tool in the construction of meanings. The sequencing of events is what we call the plot. Gergen (1998) notes that events are carefully selected to support an endpoint. This observation is critical when designing activities. The learner should be able to identify the endpoint served by every event or segment of text.

The rules of Guess my Robot are essentially a skeleton for a plot: they define a sequence of events, one logically leading to the next. Furthermore, representing sequences as ToonTalk robots transformed them into a temporal form. A ToonTalk programmer trains an animated robot to perform a sequence of actions. This robot can perform its task in such a way that one can observe its actions. The normal, timeless, form of $a_n = b*a_1 + c$ becomes a robot that *first* multiplies by *b*, *then* adds *a*, to produce the *next* element of the sequence from its *current one*. This representation of sequences, as of the rules that generate them, is much closer to the observed intuitive form that students use. Yet it is no less rigorous than the standard mathematical notation. It is also aligned with the insights of *embodied mathematics* (Lakoff & Núñez, 1997; 1998; 2000; Núñez, Edwards & Matos, 1999), which associated our innate understanding of sequentiality with our fundamental bodily and neurological nature. As the example of Rita's challenge shows, this representation of sequences enabled students to generate and analyze rules far more complex than they encounter at school.

### Joe999's robot

Joe999 is an 11-year-old boy from London. His group worked with Ken on a different activity, and was not involved in the Guess my Robot game. At some point, Joe999 started using the system's messaging facility to chat with Yishay (whom he had never met). His messages were social in nature. Yishay tried to divert the conversation to activity related content. Eventually, Yishay invited him to join the game. Joe999 found a challenge posted by Yishay, and Ken showed him how to load the box into ToonTalk and how to use the wand to copy and subtract numbers. After some hard work, he managed to solve the challenge.

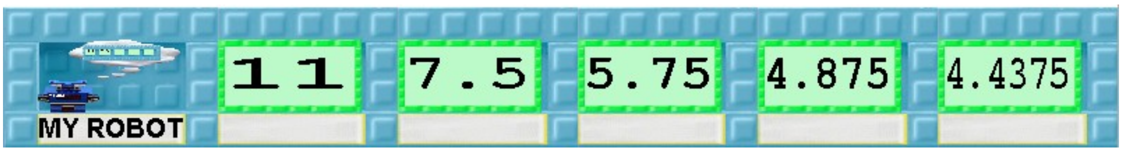
Joe999 was very proud of his achievement and was confident he could train a robot to build it but had only time to write a short comment.

> Yish. After 10 minutes I figured out how to do the sequence. You take away 3.5. Then you find half of 3.5 and take that away from 11 and continue this sequence.

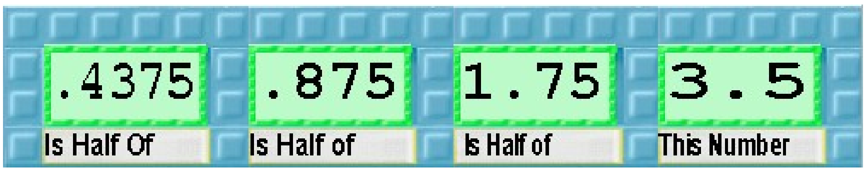
To which Yishay responded:

> **Can you explain?**
>
> Don't just talk. ToonTalk. Instead of telling my you figured it out, build a robot (or chain of robots) that produces this sequence.
>
> See you soon!!
>
> - Yishay

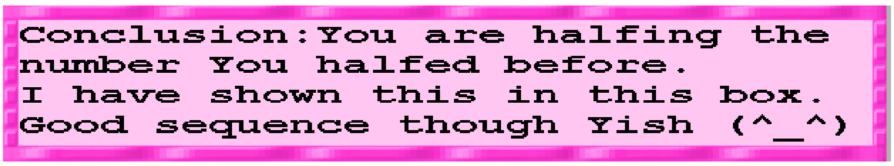
Joe999 took up the gauntlet, and trained a robot.




To our surprise, this robot did not produce the sequence – it acted out the story of how Joe999 had solved the puzzle! The robot takes the differences of the sequence and arranges them in a box:




Then it prints:




Joe999 had used ToonTalk as a narrative medium. He had turned the execution of a program into a domain-specific genre. Without any guidance from us, he had used programming as a way of making a mathematical argument. This argument was narrative in structure, yet precise and succinct in nature. It is contextualized – by the ToonTalk environment and then by the packaging of the robot; it has a plot – the robot goes through a carefully chosen sequence of actions and events; it acts as an avatar for Joe999, expressing his voice when typing "I have shown this in this box. Good sequence though Yish".

Yet at the same time, this form of expression leaves no room for ambiguity. After all, as anyone who has ever programmed a computer knows, if you are not completely accurate in your coding, the result will be anything but what you intended it to be.

Finally, Joe999's code has a moral. The purpose of the protagonist's (robot's) actions in the story is not their immediate outcome (a box of numbers, a block of text), but the implicit transfer of an idea.

## Blabs

As mentioned above, blabs are our version of the ubiquitous blog (web-log). It allows users to maintain a personal journal. Our vision was that users would use this feature to share thoughts about the activities they are involved in. Compared to the popularity of blogs in general, we were surprised by the lack of interest in this feature. We can only conjecture as to the reasons.

We believe that to a large extent the lack of success of this feature is due to the fact that we had not incorporated it into the activity design. The low participation level in blabs contrasts with the take up of other tools we provided. In other cases, we established community practices of using the tools, either through activity design or through direct interaction with learners. In the case of blabs, we created the space for practices to emerge – but alas, they did not.

A second possible source of indifference towards blabs is the lack of social interactivity. This hinders their utility in two ways: lack of motivation, and limited guidance. On the one hand, users prefer means of expression which allows their peers to react. Learners would often publish content which would naturally fit a blab item in as a short report, possibly seeking the option of a response. On the other hand, interactive elements allow researchers, teachers and peers to orient participants towards socially acceptable forms of usage, by encouraging some types of content and genres and discouraging others.

## Conclusions

In this paper we have proposed a narrative-oriented framework for design and analysis of CSCL systems and math-educational activities. The main elements of this framework are *Genre*, *context*, *voice* and *plot*. This framework was used to analyse a software system and the activities it affords. The success of the system is evaluated through the success of the activities.

We opened with three questions:

- If, and how, can mathematical meaning be expressed in narrative forms – without compromising rigor?
- What are the narrative aspects of user interface? How can interface design be guided by notions of narrative?
- How can we harness the power of narrative in teaching mathematics, in a CSCL environment?

Programming can be an intermediate form bridging narrative and mathematical meaning. Programs are sequential, require a context, and can express the style of their author. Yet they are no less valid than algebraic formulae as a means of mathematical expression.

CSCL system interfaces can support voice, multiple genres, context and plot both implicitly and explicitly: on one hand, by providing automatic markers of time and person; on the other, by providing enough flexibility for authors to express themselves in a natural, narrative manner.

By embedding narrative elements in the design of the WebReports collaborative system, utilizing the narrative features of the ToonTalk programming language, and applying a narrative-oriented approach to the design of activities we have enabled students to utilize their narrative intelligence in constructing mathematical knowledge.

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