Mathematics and E-Learning: A conceptual framework

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MATHEMATICS AND E-LEARNING: A CONCEPTUAL FRAMEWORK

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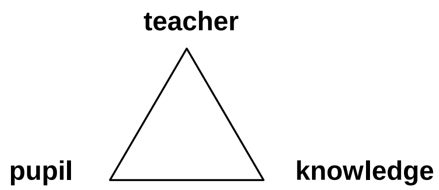
Abstract

This paper starts from the study of the epistemological statute of the didactics of the mathematics (Henry, 1991; D'Amore, 1999), which faces the phenomenon of learning from the point of view of fundaments, in order to give useful and specific considerations for e-learning environments. Investigations on how the triangle teacher-pupil-knowledge changes are presented. Then the model of a-didactic situations (Brousseau, 1997) is analysed in the context of e-learning platforms.

1. The triangle "pupil-teacher-knowledge" in didactics

During last twenty years the research in didactics of mathematics has analysed in various modes and with accurate details, what it is beyond the triangle whose "vertices" are the pupil, the teacher and the knowledge (Chevallard e Joshua, 1982; Chevallard, 1985; D'Amore e Fandiño, 2002):



According to *didactics*, it represents a *systemic model* useful to situate and analyze the multiple relations among the three "figures" representing the "vertices" of the triangle. The complex nature of the systemic model comes from considering at the same time all the mutue relations among the vertices, including various implications of different nature.

For an accurate deepening of such topic we cite the synthesis in D'Amore e Fandiño (2002). In such analysis, the triangle has not an explicative and descriptive function of the education experience but, above all, methodological: each vertex of the system is the observer from which we look at the relations betweens the others, even if we are conscious that none of the involved figures can be considered totally separated by the others.

Moreover the implicit effort is to render such scheme as more comprehensible as possible of the multiplicity of variables involved on the educational experience intended as problematic experience.

In such systemic model we can distinguish at least three categories of incident bodies:

- *elements* (that are the “vertices” or poles)
- *relations* among the elements (that are the “sides”)
- *processes* that identify the functional modalities of the system.

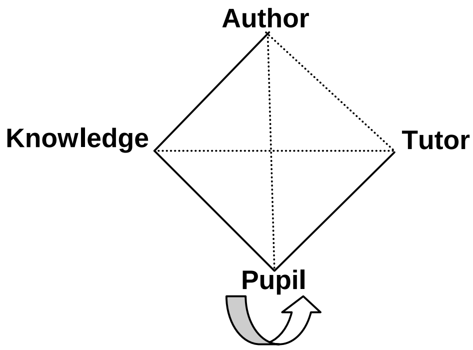
On the triangle the noosphere (Chevallard, 1992) insists, that is the external world, the society, the collection of the people which prepare the contents and the teaching methods, with their waits, their pressures , their a priori choices.

In the following a revision of such systemic model is presented (section 2), paying particular attention to the model of a-didactic situations in e-learning environments (section 3).

2. How the didactical triangle changes when e-learning platforms are used

E-learning environment can be used both in distance and blended education: the different management of the platform impacts in different ways on the vertices and on the relations among them.

The didactical triangle becomes w.r.t. such reference framework a more complex structure with new vertices and different relations. More complete vision can be found in (Albano et al., 2004a)) and an example of how some of the flowing described relations can be implemented in an e-learning environment can be found in (Albano et al., 2004b). The vertices involved in the learning process under such environment are four: *the author, the tutor, the pupil, the knowledge*. We do the hypothesis of the following structure where the introduction of the ICT has a total influence, with different levels of deepness, on the four vertices and on different arisen relations and implications:



In our opinion such scheme concerns the complex system of the relations arisen among the figures interacting in the learning process when we use a distance learning platform, defining at the same time the specificity and the problematic aspects. Of course, we point out the influence of the noosphere.

2.1 The “vertices”

• **The author.** In traditional teaching the teacher is author, tutor, evaluator of his/her course. In an e-learning environment it is possible to focus on two specific figure: the author and the tutor. The first is not yet a single figure, but with this name we mean a group of persons with different professional skills: the instructional designer/manager, the graphical expert, the ICT expert, the didactical (general and disciplinary) expert, the pedagogical expert, the sociologist, the knowledge domain expert, the communication expert… The richness of the involved figures in such pole allow to create a variegated scenario of pedagogical waits concerning knowledge, of professional or ideological beliefs, of implicit philosophies (Porlán et al., 1996) that supplies with an enrichment of the platform. We consider that the comparison, the discussion, the thoughts that can occur among the different experts above, having diverse experiences, allow to take decisions about the content (*didactical transposition*)[1] (Chevallard, 1985, 1994) to be insert in the platform and about the methodologies through which a certain content is introduced (didactic engineering[2]) (Artigue, 1992; Trouche, 2004), arriving in such way to the construction of a reach and deep product.

• **The knowledge.** We mean as knowledge the academic, official one, the result of the research, which represents the stakes of the educational system and that presents specific structural, methodological, historical and cultural characteristics. It is interesting to consider that the ICT tools allow new ways of presentation of knowledge (dynamical and interactive instruments, virtual manipulations, simulations, animations…) and new knowledge. Such knowledge will not be exclusively disciplinary anymore but they will be incremented at least with ICT and foreign languages competencies.

• **The pupil.** Nowadays the addresses of the training, besides the young people who attend full time schools and universities, will be workers, professionals, adults involved in *lifelong education* (Butera, 2002). Even in e-learning environment, according to our opinion, there is the necessity to make reference to cultural and cognitive projects that are personal as much as possible. Thus it is necessary to avoid that the platform is structured in such a way that the author is the only responsible of the choice of the significant knowledge for the pupil, fostering so the student to give up to take personal care of his/her choice according the preference, the interest, the motivation. Moreover it would be opportune to study a learning environment in order to foster the actual attention to the "fundamental stones of the foundation" of the knowledge: the actual competencies (Cornoldi et al., 1995), the waits (Elliott e Dweck, 1988), the beliefs (Nicholls et al., 1990) and the personal cognitive style of the learner (Sternberg, 1996).

• **The tutor.** Didacticians consider essential this figure in order to have an effective learning, but we need to stress that the definition is a little bit changed: we intend as tutor any teaching agent (human or artificial) able to intervene on student's learning

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[1] The *didactic transposition* is intended as the work of transforming the knowledge in object to be taught w.r.t. to the place, the audience and the didactic finalities to be reached. Thus the teacher has to do a transposition from the *knowledge* (arisen from the research) to the *knowledge to be teached* (selected by the institutions) to the *taught knowledge* (chosen by the teacher as specific object of his didactic work).

[2] The studies on *didactic engineering* concern in particular the elaboration of didactic sequences, the creation of tools and didactic material organised coherently to the reaching of specific learning objectives. Actually with such word we refer to a research methodology of qualitative type (Sarrazy, 1995; Farfán Márquez, 1997).

process (Balacheff, Sutherland, 1999). The role of the tutor impact different areas: management/organisation, social and didactical (Cosetti, Pallavisini, 2002).

2.2. The "relations"

We consider the following relations among vertices:

• **author-knowledge**, characterised by the verb "transpose", where the main activity is the first part of the didactical transposition from the *knowledge* to the *knowledge to be taught* that have to be realised trying to avoid the possibilities of misconceptions. It is fundamental the phase of projecting *a-didactical situations* (Brousseau, 1986) through which the learner constructs personally his knowledge; the proposals have not to be explicitly didactical, in the sense that the learner has to be involved in an activity but he has not to explicitly know which are the cognitive finalities to reach. Proposing a-didactical situations, the author realize the *devolution* that represents the action of the author towards the pupil so that this latter is encouraged to *involve himself* through the methodology integrated in the platform. The motivation at the basis of the implication is conditioned by the choice of the contents, how they are structured, the adopted methodology to present them, thus the phase of projecting is fundamental and it is bound to the type of Learning Object[3] supported by the platform and their organisation.

• **author-pupil** this relation has been sketched to highlight that no direct contact between these two vertices is active. Anyway some relation exists in a form of a feedback: author continuously corrects the courseware according to the information on the pupils' learning process, collected by the tutor or by the analysis of statistical data of automatic assessment. Thus we say that the relation between author and pupil is mediated by both knowledge and tutor.

• **knowledge-pupil**, expressed by the verb "to learn". The use of technologies has had a great impact on the accesses to such knowledge. The phase that characterises such relation is the *implication* that is the action of the pupil on himself once accepted the devolution, taking personal care of his own knowledge (phase that can occur if the author has foreseen a-didactical situations). The "knowledge" is so constructed by the pupil and its validity is transferred by the acceptance of the community interested

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[3] A Learning Object (LO) can be defined as *"Any entity, digital or non-digital, which can be used, re-used or referenced during technology supported learning"* (Learning Object Metadata Working Group of the IEEE Learning Technology Standards Committee - LTSC). We cite their characterisation adapted from the Wisconsin Online Resource:

- LOs are a new way of thinking about learning content. Traditionally, content comes in a several hour chunk. Learning objects are much smaller units of learning, typically ranging from 2 minutes to 15 minutes;
- LOs are self-contained – each learning object can be taken independently;
- LOs are reusable – a single learning object may be used in multiple contexts for multiple purposes;
- LOs can be aggregated – learning objects can be grouped into larger collections of content, including traditional course structures;
- LOs are tagged with metadata – every learning object has descriptive information allowing it to be easily found by a search.

to the problem by the phases of *validation*[4] and finally of *institutionalisation of the knowledge* foreseen by the author or the tutor.

• **author-tutor** is characterised by the verb “to collaborate”. In fact we wish that there will be collaboration between the author and the tutor otherwise some difficulties can occur such as not sharing of the choices, different conceptions, difficulties for the tutor to understand the methodological choices that are at the basis of the platform. Moreover the tutor’s reports from the students’ practice are very important for the author to improve the courseware. Such relation is less evident w.r.t. the others, this is why we have chosen to sketch it.

• **tutor-pupil** is represented by the verbs “to facilitate - to advice - to guide”. In fact the tutor should allow an harmonisation of the different phases of the learning process, a “help” in the didactical environment, both affective and emotional, a sort of “orchestra director” which allows the management of the times, a reference point for the choices of the pupil (w.r.t. to the contents or the personalised learning styles), an investigator and possibly a “solver” of the possible misconceptions, or false beliefs that will be evident by the doubts of the pupils… defining in such a way an “instrumental orchestration” (Trouche, 2004) trough a didactic configuration and its modes of exploitation. Generally in the platforms such figure is not appraising, so the “classical” a-symmetric situation between teacher and pupil is not so stressed, in particular when an automatic evaluation is foreseen or even if the final phase of evaluation is not foreseen or if such phase is managed by a person different from the tutor. In all such cases the *didactic contract* [5] is remarkable modified because it is no more affected by the evaluation phase.

• **tutor-knowledge** is expressed by the verb “to manage”. The tutor acts as manager of the courses: he can define a programme, homeworks, deadlines and priorities, he can insert the material he judges suitable w.r.t. the course he is creating/managing. In such relation the *didactic engineering* is included, agreed as a modality of elaboration of sequences, tools and didactic material finalised to the learning of specific contents and as methodology finalised to the organisation, transmission and acquisition of specific competencies that are object of the teaching/learning process. In this sense the tutor is engaged in the last phase of *didactical transposition* from *knowledge to be taught* and *knowledge actually taught*. According to our opinion such relation is filtered by the relation tutor-pupil, since the choices of the tutor about the knowledge

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[4] The validation is the process adopted in order to reach the belief that a certain obtained result (or a constructed idea) actually corresponds to the requirements explicitly implied; this can occur when a pupil poses himself in an explicitly communication situation, addressing his attention to the transformation of his own personal knowledge in a communication product, validating so his construction.

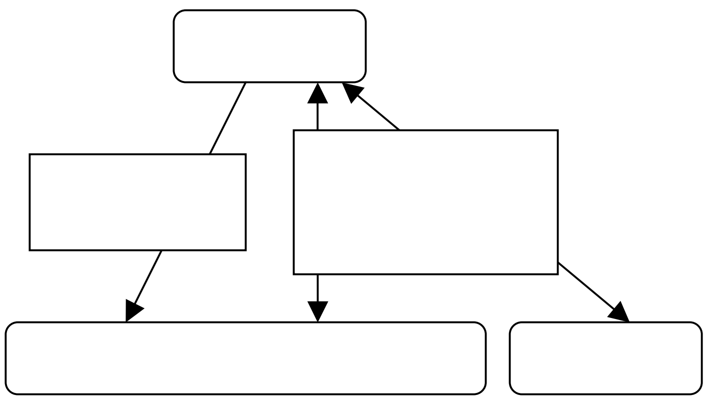
[5] The idea of *didactic contract* is the following: *«In a teaching situation, prepared and realised by a teacher, the pupil has generally the to solve a (mathematical) problem that has presented to him/her, but the access to such task has made trough an interpretation of the posed questions, of the given information, of the imposed constraints that are constant in the way of teaching of the teacher. Such (specific) behaviour of the teacher expected from the pupil and the behaviour of the pupil expected from the teacher constitute the didactic contract»* (Brousseau, 1980). The expectations that characterize the didactic contract are often due to implicit agreement depending on the conception of the school, of the mathematics …

are always addressed to the pupil, this is why we have chosen to sketch it in the picture.

• **pupil-pupil** can be represented by the verb "to socialise". Such relation assumes a greater emphasis in e-learning environment. The reason of the greater interest we give is due to the fact that in the classical didactic such relation is in some way "natural" due to the physical contact among teachers and pupils; instead the e-learning environment foresee a privileged contact with the computer, addressing principally an individual learning process, that results a solitary construction, while it is the product of a real and complex interaction with the members of the micro-society the pupil is part of. Thus the cognitive activity is a behaviour that is realised through the interaction among people: it is an inter-subjective process that is socially organised and it is specific w.r.t. a context (Cole, 1996): *«the learning process of an individual is fostered by the interaction of a group »* (Butera, 2002). So the new technologies have to support and foster the collaborative learning through communication systems, shared synchronous, systems supporting group processes. The need of the "social" factor has brought today to the born on the net of the *learning communities* and of the *practice communities* (Ligorio, 1996); in fact it is important that the pupil identifies himself in a community, even if in a virtual one.

3. A-didactic situations in e-learning environment

According to Brousseau (1997) *«In modern didactique, **teaching** is the devolution to the student of an adidactical, appropriate situation; **learning** is the student's adaptation to this situation»*. A-didactical situations seem to fit very well the e-learning environments: the student is implicated in constructing his knowledge interacting with a "milieu", properly designed by the author and the tutor in order to foster the devolution (interesting examples can be found in Laborde, 2001). We refer to various types of situation, distinguished w.r.t. the relation that may exist between a student and the milieu, according to the following model (Albano, 2004; Albano, D'Auria et al., 2004) :



• *Situations of Action*: are those in which the student interacts with the environment: *«If the exchange of information is not **necessary** for obtaining a* *decision, if the students share the same information about the milieu, the "action" is dominant.*» (Brousseau, 1997). The sequence of situations of action constitutes the process through which the learner constructs strategies, namely "teaches to himself/herself how to solve the problem. In this sense Brousseau talks of "dialectic of action" since the student on one hand can anticipate the result of his choices and on the other hand the chosen strategies can be confirmed or not by the experimentation/interaction with the environment. The situations of action promote in the student the rising of a "model", namely of a representation of the situation, which may be more or less implicit. On the basis of the model the student little by little constructs, he will do his following choices.

In e-learning environment, the student can be immerse in a "real" motivating and involving context, which foreseen some active phases and choices made and personally managed by the student, to whom the milieu replies. Such situations can be realised using "expressive tools", that can be distinguished in pedagogical tools (e.g. Dynamic Geometric Systems (DGS), microworlds, simulations) and calculation instruments (e.g. Computer Algebra System (CAS), spreadsheets, graphing calculators, databases), properly arranged by the author/tutor. Here the milieu acts as a black-box: the students changes some parameters and observes how the environment modifies.

In (Holey, Noss, 2003) digital technologies are reviewed w.r.t. their impact in mathematics education. Expressive tools give the student many advantages, such as: to manage competences greater than he actually has (e.g. to make difficult computations, to plot, to apply algebraic transformations, etc.); to have a direct and immediate feedback; to use many semiotic registers (algebraic, graphical, numerical); to concentrate his attention on qualitative aspects rather than procedures. Note that the action in e-learning environment has an added value w.r.t. the paper-and-pencil: for example a figure sketched with a DGS is not static, but through draw mode allows to outlined all the figures preserving some geometrical properties, fostering the student to make conjectures.

- *Situations of Formulation:* are those in which the student sends messages to the antagonist milieu with the intention of presenting an opinion. When the strategies are formulated, there are two strategies of feedback: one to the environment (milieu) that, once the formulated strategy has been applied, gives a response which can be positive or not; one to the other students he interacts with, who say if they have understood. The situations of formulation encourage the acquisition of explicit models and languages; if they have an explicit social dimension, we can talk of situations of communication (D'Amore, 1999).

  In e-learning environment the student is asked to make explicit the implicit model that he has built "acting", for example he/she is asked to make explicit the relations intervening between the variables at stake, to write a formula, to realise an algorithm, etc. In this sense, building a programme, by CAS as programming language at high level, allows new ways of modelling and representing mathematics. Here the milieu acts as a white-box, that is it replies by applying the received model and the student has the possibility to understand if the supposed model produces coherent results or not.

Since learning is a social construction, it is opportune that these situations are in particular situations of communication: the explicit models of each student can be shared and discussed with other students during virtual debates, forecasting a confrontation, in a collaborative learning process, through tools (synchronous and asynchronous) specific for the communication, the sharing of the resources, to support group processes (chat, videoconference, shared work on the same files).

Situations of Validation: are the situations in which the messages exchanged with the milieu consist in assertions, theorems, demonstrations, both sent and received, namely the affirmations must be subjected to the judgement of the interlocutor who must be able to give a feedback, to protest, to reject a reasoning, to express some counter-examples, etc. The student is required to justify his assertions, to test their validity in a more formal and general manner than the simple observation of the results produced by the model implementation. In this phase an important aspect concerns the debate with the other students. These situations have to lead the student to evolve and revise his opinion, replace false theories with true ones, to organise the demonstrations. In this sense CAS used to verify generalisations, supporting students in making sense of their algebraic generalisations at a semantic level. In mathematics proof can be produced using expressive tools: attention should be given to new kind of proofs, such as those ones based on the use of logical value of algebraic operators and on the use of graphs.

Moreover, in e-learning environment a virtual area can be organised (such as a discussion forum) asking the student to produce and share a document with his models and proofs. The debate with other students is considered essential: each student has to “contest” the proofs given by the others and defend his own theses.

As pointed out in (Holey, Noss, 2003), since most of the students interacting with digital technologies spontaneously articulate justifications of their actions along with explanations of why their actions produces the expected feedback (or not), such technologies might give the opportunity to produce a deep understanding of the topic, although we have to take also into account the obstacles that might be arisen (Drijvers, 2000).

Once the students has completed the described process, the institutionalisation allows the passage from *knowing* (as personal construction) to *knowledge* (as a socially shared construction).

4. Conclusions

In this paper we have analysed how the learning process might change in e-learning environment, w.r.t. to the involved actors and the relations among them. In particular we have considered the implication of the student, that is the interaction – relation between the learner and the knowledge, structured through situations of action, formulation and validation, which lead the student to the construction of his own knowing. The impact of digital technologies has consequences as both new actors (e.g. author) and new meaning of existent ones. Moreover they seems to well fit the a-didactical situations, because they, suitably arranged by the author-tutor and as powerful tools containing knowledge, naturally foster exploration, conjecturing, explanation, verification and proof. The counterpart is represented by new obstacles that might be arisen that requires new pedagogical contexts.

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