Two Years Of Use Of The Aplusix System

Denis Bouhineau, Jean-François Nicaud, Hamid Chaachoua, Marilena Bittar, Alain Bronner

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Denis Bouhineau[1], Jean-François Nicaud[1], Hamid Chaachoua[2], Marilena Bittar[3], and Alain Bronner[4].

[1] Université de Grenoble 1, IMAG-Leibniz/MeTAH, France
E-Mail : Denis.Bouhineau@imag.fr Jean-Francois.Nicaud@imag.fr
[2] IUFM de Grenoble, IMAG-Leibniz/MeTAH, France, E-Mail : Hamid.Chaachoua@imag.fr
[3]University of Campo Grande, Brazil, E-Mail : marilena@dmt.ufms.br
[4]ERES, LIRDEF, IUFM de Montpellier, France, E-Mail : Alain.Bronner@montpellier.iufm.fr

http://aplusix.imag.fr

ABSTRACT

APLUSIX is a learning environment for helping students to learn algebra. This system is designed and developed in the IMAG-Leibniz laboratory. Its basic training mode consists of letting the students perform their owns calculations, thanks to a two-dimensions editor of algebraic expressions, providing feedback on the correctness of the calculations and on the end of the resolution.
APLUSIX has now been used for two years at school, in different contexts. One use was made by four teachers during two entire school years in France, in the computer room, for each algebraic activity of their grade 10 classes. Their main outcomes were very positive. APLUSIX has also been used in the hospital for children having a long illness. The results were very positive too. Several experiments of Aplusix have been realised with a pre-test and a post-test to measure the learning. One experiment, in France, with 28 students and 50 minutes training, shows an improvement of the abilities increasing of 70% to 250% depending of the type of exercise. Another one, in Brazil, with 1120 students and 25 minutes training, shows a global improvement of 30%.
These good results led us to realise a version of APLUSIX for home usage in addition to the version for schools. This home version is now available as a shareware in several languages. The school version will be commercialised at the beginning of 2005.

KEYWORDS : ALGEBRA, EQUIVALENCE, EXPERIMENT, FEEDBACK, ITS, LEARNING, MICROWORLD, TEL.

1. INTRODUCTION

In the domain of Technology Enhanced Learning (in short TEL), most of the work done by researchers stay confined inside the laboratories and universities, those works almost permit the development of the academic knowledge but most of them decayed without having been used by everyday students in standard school. Fortunately, it is not always the case. Few works have been exported out of the research area into the real world. The best example is given by the success story of the microworlds for dynamic geometric leaded by Cabri-Géomètre (Laborde, 1989).
In the domain of algebra, no success as big has been noticed yet. In the last ten years, almost two academic projects have been commercialised (Beeson, 1996 and Koedinger, Anderson & Hadley *et al*. 1997). Unfortunately, they did not compete with the use of computer algebra systems (in short CASs) available on the student's symbolic calculators. CASs and spreadsheet are use even if they have not been built with any learning purpose. Teachers and researchers in education have shown that CASs are certainly not neutral and safe objects, and, moreover, the use of those tools can introduce important difficulties for the learning of algebra (Trouche & Guin, 2002). For example, in the same direction that, years ago, it has been shown that calculators lead to annihilate the differences between large integers and floating point numbers (because of the scientific notation), between the decimals, the rationales, the algebraic numbers and the true real number (because of the truncated decimal representation), CASs tend to make disappear the differences between one given expression, the same expression factored and the same expression expanded, reduced and ordered. For all of these expressions, the student can find zeros, limits for infinite values, derivatives, etc., with the same action, not depending on the form of the expression.

In 2000, our team, mainly two computer scientists, the two first authors (who, at this moment, were at the University of Nantes, France), restart from scratch the development of a new TEL environment with two main goals. The first main goal was to create a TEL environment for learning algebra allowing the student to freely build and transform algebraic expressions. The environment should provide epistemic feedback that can help in the first stages of the learning of algebra. At that time, there were no TELs for algebra that allowed the student to freely build and transform algebraic

expressions, as he/she can do on paper, and to follow his/her own algebraic reasoning; the existing TELs were, and most of them are still, command-based systems that did not allow the student to proceed without applying a command chosen from a menu. See MathPert, (Beeson, 1996) as a good example of these command-based systems. So we decided to build a system to help students solve exercises in numerical calculations and formal algebra, where they would perform their own calculations by typing the expressions and making the steps they want, as on paper. The system, called APLUSIX (this is what is pronounced in French when you read "a+x"), would gives feedbacks, mainly whether the calculations are correct or not, and whether the exercise is solved or not, those feedbacks depending on the semantics of algebraic expressions, and the syntactic form of the expression, see (Nicaud, Bouhineau & Gélis, 2001) for a complete explanation of the semantics aspects.
The second main goal was to create a TEL environment that would be used for real. This goal requires building an easy to use and useful system. We considered two main points for usefulness: Encompassing a large mathematical domain and having several modes of functioning. And we also kept in mind that teachers will be the first users of the environment. For them, also, the environment must be user friendly and must provide useful tools.
APLUSIX does not directly teach the rules and the methods. Rather, by providing adequate information, it helps students learn to apply the rules and methods correctly. The teacher is still in-charge of the teaching and learning process, for him/she APLUSIX is just another tool in order to achieve his/her goals. In fact, APLUSIX impacts the learner rather than making the teacher more efficient. Aplusix participates to the global movement observed about education and TEL: computers are slowly but steadily moving educational programs from teaching to learning.

Section 2 will give a short description of the system. Section 3 will describe experiments done by researcher in education with it. Section 3 will report comments given by teachers having use APLUSIX during the last two years. Section 4 will relate the efforts done to achieve a real distribution of the system.

2. THE APLUSIX SYSTEM

APLUSIX is organized on the base of two editors, an advanced editor for algebraic expressions, an editor for algebraic reasoning; these two editors have been constructed as microworlds. They permit numerical calculations and formal algebra activities like expansion, factorisation, and resolution of equations, inequations, and systems of equations with verification of the equivalence (or equality) of expressions during the reasoning. There is an immediate feedback showing whether two consecutive expressions are equivalent or not.

The domain allowed correspond to numerical expressions with integers, decimals, fraction, square roots; algebraic expressions with degree less or equal to 4 (polynomials or rational expressions with sum of degree less or equal to 4); equations, inequations or systems of equations.
Strategic information is given to the students about the state of the problem in order to help them move toward a solution. Gauges 'Reduced', 'Sorted', 'Expanded', 'Factored' (for numerical and polynomials factors), 'Equation' are available.

Commands for executing certain algebraic actions are available. These commands can be visible or not, powerful or not, according to parameters set by the teacher, (they have to be adapted to the current level of understanding of the students in order to only present calculations they can do without difficulty). With this feature, such a computer system can provide an introduction to the proper use of a Computer Algebra System. There are 'Calculate', 'Expand and reduce', 'Factor' and 'Solve' commands.

Last, the microworlds are embedded in a piece of software whose concern is the practical organisation of the learning process (login of the students, organisation of their work according to some prepared scenario, recording of the activities, scoring and statistical analysis of these activities made available for the teacher, tuning of the software with parameters, automated generation of exercises, ...). A longer description is given in (Nicaud, Bouhineau & Chaachoua, 2004).

Let's see an example of use of APLUSIX. First, the student starts with an expression (14 - 8)(18 - 15) and a direction 'Calculate', figure 1.

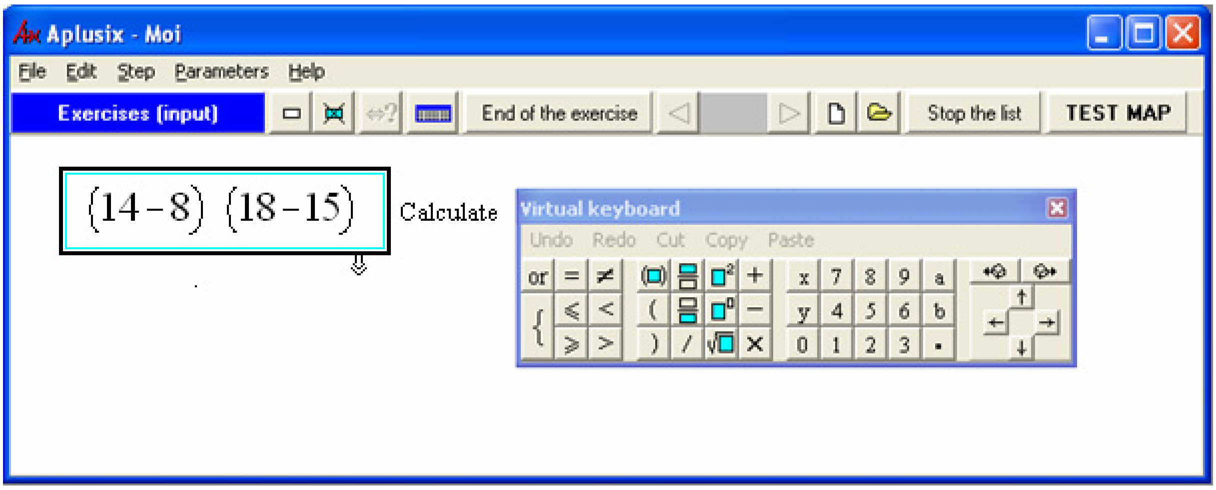

Figure 1: Starting point of a session with APLUSIX.

Then, figure 2, the student, tries to calculate the product, see the negative feedback, correct him/herself, and make a significant step.

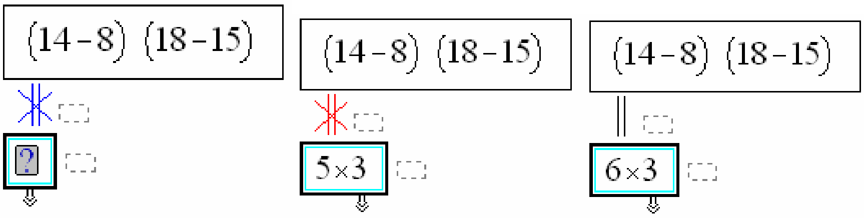

Figure 2: Three moments in the calculation. First the student ask for a new step (no equivalence is shown in blue, the colour of the question mark, between the first step and the second unfinished one), then the student proposes an erroneous expression (a negative feedback is given in red), after correction, the student finds the correct expression.

3. THE EXPERIMENTS

The first users of APLUSIX are our colleagues from math education. They organize large experiments in ecological conditions to record via APLUSIX the activity of students with this light software very close to the paper and pencil environment (especially when no feedback is given to the students, as it can be parameterised in APLUSIX)

3.1 Experiment '9-2'

Quantitative information:

- When: September and October 2002
- How many sessions: 3 sessions, the first session without any feedback from the software, the second session with feedback on equivalence available to students when they ask for it, the third session with permanent feedback.
- How much time: 50 minutes for each session.
- How many students: 246 students, three levels involved: 91 students aged 14-15, 108 students aged 15-16 and 47 students aged 16-17.
- What were the exercises: Each session, and each level get a particular list of exercises from every type of problems available in APLUSIX. For example, there were the following exercises: expand (7x-2)²-(3x+12)(3x-12), solve (3x+5)(-x+9)=0, solve (-3x+9)(2x+4)-(-3x+9)(7x-1)=0, solve $x^2 = 3$, solve $\frac{(2x+4)(5-3x)}{4x(x+1)} = 0$, solve $\frac{2x-2}{5x-3} = 1$, solve 6x-(5-3x)<-3(x+1).
- What is the average score (in terms of well-solved exercises) obtained on exercises: it was about 30% for 14-15 years old students, 40% for 15-16 years old students and 60% for 16-17 years old students.

Qualitative information:

The goal of the experiment '9-2' was to collect an important amount of protocols in order to analyse and determine typical conceptions of students. Each list of exercises consisted of about 30 exercises organised to minimize the effect of learning. For example, the first session, for 15-16 years old students was made of 3 'calculate' problems interleaved with 8 'expansion' and 6 'factorisation' about second degree polynomials, and 13 'solve' problems about linear, second degree equations and systems of linear equations. Most of the numbers were integers. However, in 5 problems, they were rational numbers and radicals in 1 problem.

Two different analyses of the results have been made by hand:
- Construction of a collection of correct and incorrect transformation rules used by student. This has been made by means of an analysis by hand of the protocols. From an epistemic point of view, regarding the movements of terms in linear equation and inequation, there can be found 648 transformation rules. The library of transformations rules used in algebra allowed the construction of an algorithm to diagnose the transformation made by students. This algorithm permitted the second analysis which aim at:
- Construction of typical conceptions of students learning algebra. The algorithm for diagnostic of transformation gave good results, but we have not be able to construct typical conceptions of student, because we did not get enough data on one domain for every student. The principal reason was that we have targeted a too large domain (too many types of problems and sort of expression). As a consequence, we decided to organize a second experiment, where the domain (type of problems, type of expressions) would be smaller, in order to get more actions for each student on one domain.

3.2 Experiment '9-3'

Quantitative information:
- When: September and October 2003
- How many sessions: 3 sessions, the first session without any feedback (pre-test), the second session with punctual feedback on equivalence available to students, the third session without any feedback (post-test).
- How much time: 50 minutes for each session.
- How many students: 167 students aged 16-17.
- What were the exercises: the exercises were mostly linear equations and inequations. For example: solve 2x+4=7+5x, solve 3x+2<6x+11, solve 5(x-2)-(1-x)=0, solve (x+1)(x+2)<(x+3)(x+4). The exercises of the third session were close variations of the exercises from the first session (5(x-2)-(1-x)=0 replaced by 3(x-2)-(5-x)=0)
- What is the average score obtained on exercises: we pay attention to the evolution of the score between pre-test and post-test for 28 students from one high school (the score is the number of exercises well-solved by all the students over the number of students). On the average, this score have been up of more than 100%:

Exercise of pre-test	Similar exercise of post-test	Score at pre-test	Score at post-test
3-2x=3x-6	2x+4=7+5x	37%	64%
3/2+x=1/3-1/2	3/2+1/2x=1/3-1/2x	10%	32%
-4x-1>0	-4x-1>2x	16%	42%
3x-8<2-2x	3x-8<2-2x	33%	66%

Figure 3: Evolution of the score between pre-test and post-test.

Qualitative information:
The goals of the experiment '9-3' were to collect protocols about a restricted domain of algebra to analyse, and to determine typical conceptions of students on this domain. As a derivative result, we would like to obtain also some preview about the learning achieved with APLUSIX. The restricted domain was about solving linear equations and inequations. We have focused on the transformation rules of movement in linear equations and inequations. For example, about the transformation 3x=4 → x=4/-3, we have associate the rule 'MultiplicativeMovementInRelation' applied to term '3' with attributes (LeftToRight, NumeratorToDenominator,SignArgumentChanged, Incorrect)

Protocols have been processed by the algorithm to diagnose the transformation rules used by the students. Among 4570 resolutions, we have diagnosed 4756 transformation rules concerning movement. This has produced vectors containing information about the context and the transformation rules used. Those vectors can take 192 different values. For each student, we have tried to correlate context and transformation rules in order to determine the conception of the students about movement in linear equations and inequations. We have found 4 conceptions based on the sign of the moved terms, and 4 conceptions based on the sense of the inequations.

3.3 Experiment 'BRA-1'

Quantitative information:
- When: March, April 2004
- How many sessions: 2 sessions. Each session began with permanent feedback on equivalence (25 minutes), and ended with no feedback (test during 25 minutes).
- How many students: 1121 students aged 13-14.
- What were the exercises: the tests consisted of 20 exercises on linear equations and inequations. Coefficients were mostly integers. Few exercises contained rational numbers.

- What is the average score obtained on exercises: the first test had only 10,7% of success on equations and only 8,9% of success for inequations. The rate of well solved was up 30% for the second test. From a general point of view, the scores were very low. This was surprising because the resolution of linear equations and inequations were supposed to be known at that age.

Qualitative information:

The goal of the experiment 'BRA-1' was to test the use of APLUSIX for learning with a very large number of students. During the first session, the first part, with permanent feedback, corresponded to an introduction to the system. During the second session, the first part was thought of as a learning phase (25 minutes). In both sessions, the second part, without feedback, was a test: pre-test for the first session, post-test for the second session.

Unfortunately, the length of the experiment was not sufficient to conclude that any learning occurred for students. Nevertheless, we have verified an increase of the student's scores between the two sessions. The observed improvement was not related to one particular type of exercise but related to the global set of exercises. For example, we have noticed a score of 25% of success for the exercise X/3=4 from test 1, and a score of 30% of success for the exercise x/7=2 from test 2. It means a noticeable improvement. But for the exercise 4x-6=2x+5 from test 1, we had a score of 21% which did not evolve with test 2: we got a score of 22% of success for the exercise 8x-2=4x+3 from test 2.

For both tests, statistics have shown that the score for equations is quite twice the score for inequations: 27% for equations, 14% for inequations. This is a normal. Equations have been studied by students during the previous two years. Equations are worked out all along these two years, either as exercises, either as problems leading to equations. Inequations did appear at the same time but the frequency is very low, even null in some manuals (equations do appears always in manuals)

3.4 Experiment 'BRA-2'

Quantitative information:

- When: March, April 2004
- How many sessions: 2 sessions. The first one, without feedback, began with an introduction to APLUSIX and ended with a test on linear equations (30 minutes). The second session began with a test on linear inequation (20 minutes). The end of the second session was free, students had permanent feedback.
- How many students: 1340 students aged 13-14.
- What were the exercises: the tests consisted of 16 exercises on linear equations and inequations. Coefficients were always integers. For example: 3x-5=7, 4x-6=2x+5 and -2(5x-2)>-3(x+2). Exercises relied mainly on numerical calculations, expansions and reductions.
- What is the average score obtained on exercises: the average score (all protocols evaluated together) is very low, varying from 3% to 26% according to the exercise. Statistics computed by high-school can vary from 35% to 60%. Score for equations are higher than for inequations. Score for equations and inequations is lower with negative coefficient for x.

Qualitative information:

The goal of the experiment 'BRA-2' was to collect a large number of protocols to continue the study of transformation rules and conceptions of students about linear equations and inequations.

Teachers reported that students did not have difficulties with the software but with the proposed exercises.

The analysis of the protocols is not finished yet. Researches are in progress. The by-hand analysis of a few protocols has shown that the incorrect transformation rules used by students correspond to the rules found during experiment '9-3'. This validates the library of transformation rules constructed. The study of these protocols in term of protocols has not been done yet. We project to use the diagnostic algorithm to achieve this study; in particular, because of the large amount of data, a by-hand study is not thinkable.

4. TEACHER'S OPINIONS

In parallel to experiment done for research, APLUSIX have been used by voluntary teacher. Here are some opinions and example of use of APLUSIX.

4.1 Use in normal class

Here are collected opinions from 4 teachers having used APLUSIX during the last two years.

About the software:

- The discovery of the system did not present any difficulty and was very quick.
- The use of the system on the network of the school is appreciated because it permits individual and collective use.
- The possibility to set the parameters of the system according to the goals of the learning is appreciated.
- The use of the virtual keyboards is more and more efficient.
- The possibility to supply the students with commands like 'expand or 'factor out' permit to focus the work on one difficulty at a time.
- The preparation of a session is easy, the exercise editor allow to create a personal list of exercises quickly.

About the students:
- The students needed less help, were more confident and gained autonomy.
- All the students really worked during APLUSIX sessions, contrarily to the usual paper-pencil sessions.
- Students in difficulty have shown motivation to solve exercises, on their own, out of the lesson; they chose exercises in their text books and solved them with APLUSIX.
- How many times do I have eared "Now, that's it, I have understood!"
- APLUSIX forces students to correct themselves their errors and to analyse them. As a consequence, the most important part of the group progress well, for the methodology too.

About the place of the teacher in the classroom:
- With APLUSIX, the teacher can circulate much more easily, bring a punctual help and give advices.
- With paper and pencil environment, the teacher mainly validates the results.
- With paper and pencil environment, the teacher spends a lot of time with students in difficulties to find where the errors are located.

About the learning:
- We have observed a significant progression from the very first hour.
- The benefit of the work with APLUSIX was transferred to the paper-pencil context.
- Progresses are made at the syntactical level.
- After two sessions devoted to systems of equations (Aplusix obliges the students to make steps with equivalent systems), 90% of the students wrote equivalent systems in the paper-pencil environment.
- At the end of the year, the students had better results than the students of the previous year.
- Some errors seem to have disappeared.
- The notion of equivalence seems to have gained sense.
- Students remember to simplify the results.

Example of use:
- Experiment and confrontation of different methods for the resolution of the same problems are facilitated.
- Each student has a computer, first with permanent verification of the equivalence, then with verification on demand.
- Each student has a computer and a list of problems, students can choose the computer or paper and pencil.

4.2 Use at the hospital and at home, for children having long illnesses

APLUSIX have been used for ill children, during one year, by one teacher, in two main situations:
- Student, in a classroom, who alternatively takes a lesson (alone or in a group of 2 or 3) and work in semi-autonomy;
- Isolated student, at home or in a sterile room of the hospital, who alternatively take an individual lesson (by teleconference: the use of Aplusix is shared between the teacher and the student) or work in autonomy.

The student has often less time to work than a usual student, and the duration of its stay in the hospital is not always known in advance. One has to do simple and efficient actions.

Up to now, the use of APLUSIX at the hospital is basic: a brief explanation of the functionalities allows an immediate use of APLUSIX, generally in anonymous mode, to work on numerical calculations (fractions, square roots), algebraic calculations or equations. The first exercises are solved with the presence of the teacher (physical or at distance) who provides the exercises, and quickly the student works in semi-autonomy or autonomy with exercises provided by the teacher or by its own initiative.

Some features of the software are particularly interesting for our specific students:
- Lightness: the discovery is very quick the manipulations are simple for the students (the virtual keyboard is appreciated for the laptop we lend to the students);
- Efficiency: Aplusix saves the teacher time; it favours the student's autonomy, promotes an exploratory behaviour, asks interesting questions (0.33 is refused for 1/3 and 1,414 is refused for square root of 2);
- Attraction: the students like to use the system, even those who would not have been motivated by the same exercises on paper;

- Flexibility: the system is easy to use in an individualized way or at distance;
- Reliability: its rigour and logic allow letting, in confidence, the student work alone.

5. TOWARD A REAL DISTRIBUTION OF THE SYSTEM

These good results led us to achieve our project of real use and to define a version of APLUSIX for home usage in addition to the version for schools, and to try to commercialize both versions. We present shortly modification we have made recently to achieve both versions and work necessary to become a decent product.

5.1 APLUSIX at home.

Compared to the software used during the experiments and last years, the version of APLUSIX tailored for home usage has been:
- simplified (fewer menus, parameters, and functionalities, smaller domain of application),
- augmented with a generator of exercises,
- augmented with an algorithm to score the production of a student.

Since, September 2004, APLUSIX is commercialized as a shareware on the web.

5.2 APLUSIX at school

The development of APLUSIX continues with a focus on the tools we can give to teachers.
We added a new type of problem: the word problem (to find an equation modelling a problem given in natural language).
We added tools for the administration of the student's accounts, of the classes, of the teacher's teams.
We added a module for doing statistics about the actions of the students.

5.3 Other works

We have tried to have APLUSIX become a professional product, with help according to the HLP format of windows, installation setup according the standard of windows. We added key-authentication algorithm to verify that the product have been paid. We have registered APLUSIX at the APP (Agency for the Protection of Programs). We have found a publisher to commercialize APLUSIX in French. We are discussing with publishers for English, Portuguese and Japanese. We also put APLUSIX on the web at http://aplusix.imag.fr .

Our opinion is that all the work done in this subsection (except the diffusion on the web) is not a research work, but is necessary to achieve a real use.

6. CONCLUSION

The work we began in 2000 has given software which can have been used since 2002. The first users were researcher in education. They collected hundreds of protocols (more than 1 millions of actions have been recorded and are available in a database not described in this article, dedicated to the protocols of APLUSIX). For researchers in education, APLUSIX is a tool which permits to obtain a lot of data. In fact, experiments have given much more data than researchers can exploit since they got it.

Since 2003, APLUSIX is also available on the web, and more than 5000 downloads have been processed (in three languages) With the exception of the few teachers we know and work with us, it does mean that there are teachers, we don't know, who are using APLUSIX in their class. In Italy, in Canada, researchers begin to work with APLUSIX, out of our control, too. We hope this will continue and progress.

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