Problem-solving and Web resources at tertiary level

Claire Cazes, Ghislaine Gueudet, Magali Hersant, Fabrice Vandebrouck

PROBLEM SOLVING AND WEB RESOURCES AT TERTIARY LEVEL

Claire Cazes, *LUTES, Université Paris 6 et Equipe Didirem, Université Paris 7*
Ghislaine Gueudet, *IUFM de Bretagne et CREAD*
Magali Hersant, *IUFM de Nantes et Centre de Recherche en Education Nantais*
Fabrice Vandebrouck, *IUFM de Versailles, Université d'Evry Val d'Essonne et Equipe Didirem, Université Paris 7*

Abstract

We organised two experimental teaching designs involving web resources in two different French universities. In this paper, we describe these experiments and analyse the students' behaviours. Our aim is to observe whether the use of specific online resources favours the development of problem-solving activities.

I. Introduction

The use of computer resources in the teaching of mathematics at the university in France has been institutionally promoted for several years. It led to the production of several softwares and associated teaching designs. We do not intend to make a general study of the use of computers in the undergraduate mathematics curriculum. We are only interested in a special kind of internet resources, belonging to the category of online courses. More precisely, the softwares we study have the following characteristics:

- an important part of them is dedicated to mathematics exercises ;
- there is at least one classification of the exercises available (according to topic, level of difficulty, key-words…);
- they propose, for a significant number of exercises, an associated environment (it can comprise hints, correction, explanation, tools for the resolution of the exercise, score, but also corresponding courses…).

We have chosen to examine such products, that we will call "exercises' directory" because of the importance of the problem-solving activity for the learning of mathematics (Schoenfeld 1985, Castela 2000). Previous studies indicate that the work with computer leads the students to a better involvement, increases their motivation, and allows them to work at their own pace (Ruthven and Henessy 2002). But there is for the moment no evidence that the computers can help the students to develop a real problem solving activity, far from a simple drill and practice. As Crowe and Zand (2000) write: «what is undoubtedly lacking is proper evaluation of use, for there is often a serious mismatch between what the teacher intends, and what the student actually does.» (p.146). We intend here to study precisely students' behaviours, in order to answer to the following question: "Can an exercises' directory, with an appropriate associated setting, lead the students to one real problem-solving activity?". We call a "real problem-solving activity" the search by the student of a personal solution. In that process, the student governs him/herself the mobilisation of the necessary knowledge, he/she makes attempts on his/her own, even if a complete solution is not found alone. Only particular exercises can lead to such an activity. There must be no indication of method within the text, and not too many intermediate questions that split the task into elementary steps. But proposing such a text is naturally not sufficient to lead the students to a real problem solving activity. For example, if the exercise is too difficult in regard of the student's knowledge, he/she may simply remain stuck. It is then necessary to propose help, but they must be thoroughly controlled to maintain the possibility of personal search.

The results we will state stem from two experiments, conducted in two different French universities. In section II we give a synthetic description of these experiments, with the main features of the resources involved and of the teaching design, but also with two particular exercises that we retained to observe in detail the students' behaviours. In section III we describe and analyse the students' activity, using specific characteristics of it for each case. We will go back to our initial question in the conclusion (section IV). The observations presented in section III lead us to propose a balanced answer to it.

II. The two experiments

In that section we present the two experiments that led us to the results stated in section III. These experiments are named after the softwares involved: Braise[1] and Wims[2]. Both took place in 2003, with first year students following mathematics major. We describe the two softwares we used, the associated setting and the exercises that we will examine in detail. More than a mere description, we want to emphasise here the use of the grids we elaborated. These grids (presented with more details in Cazes, Gueudet, Hersant, Vandebrouck (2004)) constitute a first step into the study of a teaching design involving such a software.

II.1 Characteristics of the web resources

Table 1 stems from the general grid of the characteristics of a courseware. In the building of this grid, we firstly used more general tools for analysis of web resources, like the one elaborated by Hu, Trigano and Crozat (2001). We did not find in the literature any grid taking into account precise didactical aspects. So we progressively added to the general grid stemming from the work quoted above more didactical categories that appeared relevant for several resources. We finally tested the grid be using it to analyse different products concerning different school and university levels. However, it will probably still evolve, at least because the products themselves evolve very quickly. For the sake of brevity, we only retain here the most salient elements, for our research questions, of the two softwares we used. We do not

[1] Rationale basis of mathematics exercises, http:// tdmath.univ-rennes1.fr
[2] Web interactive multipurpose server, http://wims.unice.fr/wims, developped by Xiao Gang.

examine their mathematical content, but only their structure, with a double point of view: the didactic structure and the technical features.

Table 1: Grid of the characteristics of the resources

	Braise	Wims
Didactic choices	Problem solving	Practising several times on similar exercises, proposed with random elements.
Public	Undergraduates	All levels
Environment of an exercise	For all exercises: courses, descriptions of methods, hints, detailed solution. Summaries of the important points	Depending on the exercise: numerical calculator, computer algebra system (CAS), graphing tools…
Organisation of learning	The students work on the exercises. They access the courses only through the exercises.	The students access to worksheets prepared by the teacher. They can also access freely and solve exercises on topics they know.
Classification of the exercises	Key words: level of difficulty, theme, task. Possibility to avoid specific difficulties.	Search by key words of the theme. The direct search can not really be done by the student; it is a tool for the teacher, to elaborate his sheets.
Random elements	No.	Random elements (numerical values, but also questions) which change at each attempt.
Kind of answers awaited	The answer must be written on a paper and then compared with the solution proposed.	Numerical value or brief mathematics expression.
Feed-back	None.	"Right" or "Wrong"
Marking	No mark	Mark from 0 to 10 for each exercise. The students are supposed to make several tries in order to improve their mark.
Record of the students' activity	Log files giving some details upon students' activity.	

These exercises' bases are very different. One crucial point is the kind of answers awaited, and thus the feed back proposed. It corresponds in fact to different didactic choices. Braise proposes mathematical problems. The answer can not be simple and thus could not be interpreted by the computer. So there is no reason to offer the possibility to write it on the computer. Wims is built to encourage the students to practice several times on similar exercises. For that reason, there is a mark intended to motivate the student to make several attempts on similar exercises to improve his/her mark. It is thus necessary that the student provides a simple answer, which can be interpreted by the computer. It does not prevent some of the exercises from being really difficult. Most of them are at least quite uncommon, because of the CAS and graphing tools proposed.

II.2 Settings associated

We present the settings associated with the use of the two softwares, using again tables stemming from a general description grid of the characteristics of a setting. We distinguish two main axes in that grid. The first is the place of the computer sessions within the teaching design; the second is the role of the teacher and the students during the computer sessions.

Table2: Place of the computer sessions within the teaching design

	Braise	Wims
Mathematical content	Sequences	Calculus
Public	First year – mathematics major	
Proportion of computer sessions	2 sessions over 24	1, of 3, sessions per week for 8 of the 12 weeks
Link between computer and traditional sessions	Synthesis session after the computer sessions.	The computer sessions take place after the corresponding course and tutorial.
Assessment	No specific assessment. A traditional paper and pencil exam.	Half of the final mark is provided by the work on the computer.

Table3: Role of the teacher/students during the computer sessions

	Braise	Wims
Written notes awaited	Logbook; a similar exercise must be written for the following session.	Nothing
Number of students on a computer	Work by pairs (17 students in the whole class).	Work by pairs (36 students in the whole class).
Possibility of work online outside the sessions	Yes	
Role of the teacher	Individual help	
Use of the log files by the teacher	None	
Other	Use of the logbooks by the teacher for the synthesis.	Preparation of worksheets before the sessions.

These settings are very different; they are in fact strongly linked with the final assessment associated. With Braise, the students prepare a traditional assessment, pencil and paper way. Thus they must be prepared to write detailed solutions and proofs. Both the software and the setting are adapted for that aim. In the Wims experiment, the work on the computer intervenes in the assessment. Thus it leads to more numerous computer sessions, and it explains the choice of no compulsory written notes during the sessions.

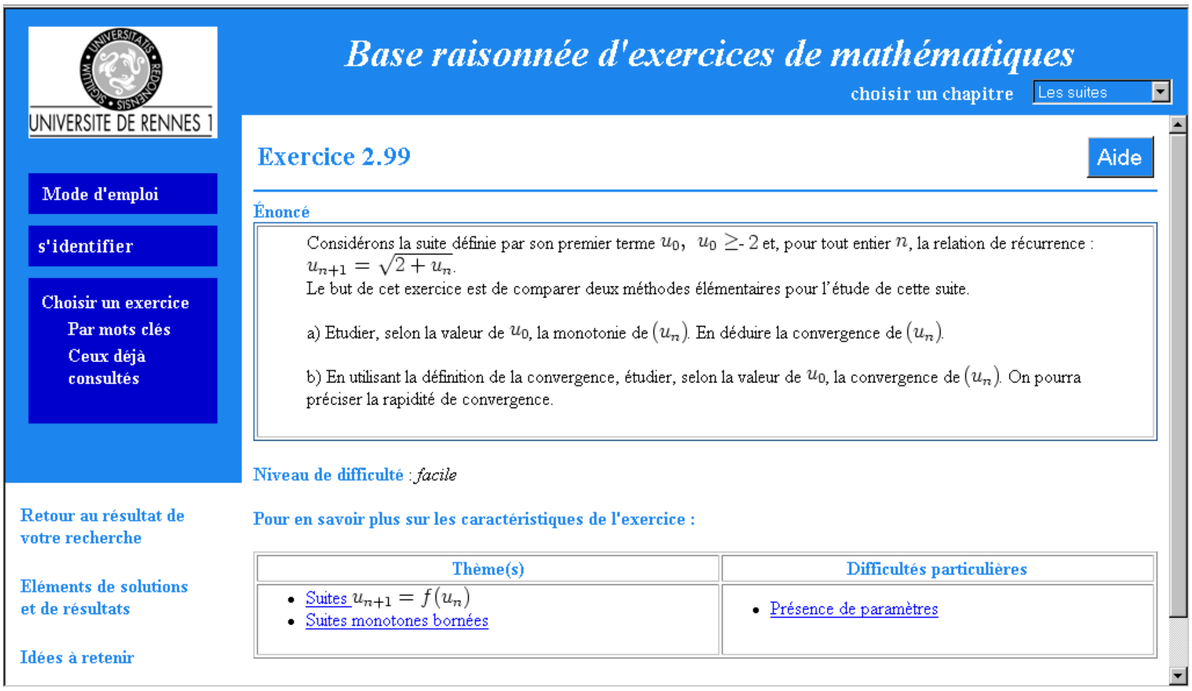
II.3 Specific exercises of each experiment

In each experiment, we will examine in detail the students' activity on a specific exercise. It is indeed necessary to study precisely the students' behaviour to observe if the work on the computer can lead to a real problem solving activity. It depends on the way they use the computer, that we will discuss in the next section; but it also depends on the exercise proposed. Do they only need to apply well-known results, or do they have to produce a personal endeavour, that requires adaptations, mobilisation of similar situations…more specific of a real problem-solving activity (Robert 1998, Robert and Rogalski 2002).

Braise

The exercise we focus on in Braise (exercise B) belongs to the theme "sequences $u_{n+1}=f(u_n)$ ", with the level "easy", and the task "determine the nature of a sequence". Figure 1 displays the corresponding screenshot.

Figure 1: Screenshot of Braise – Exercise B



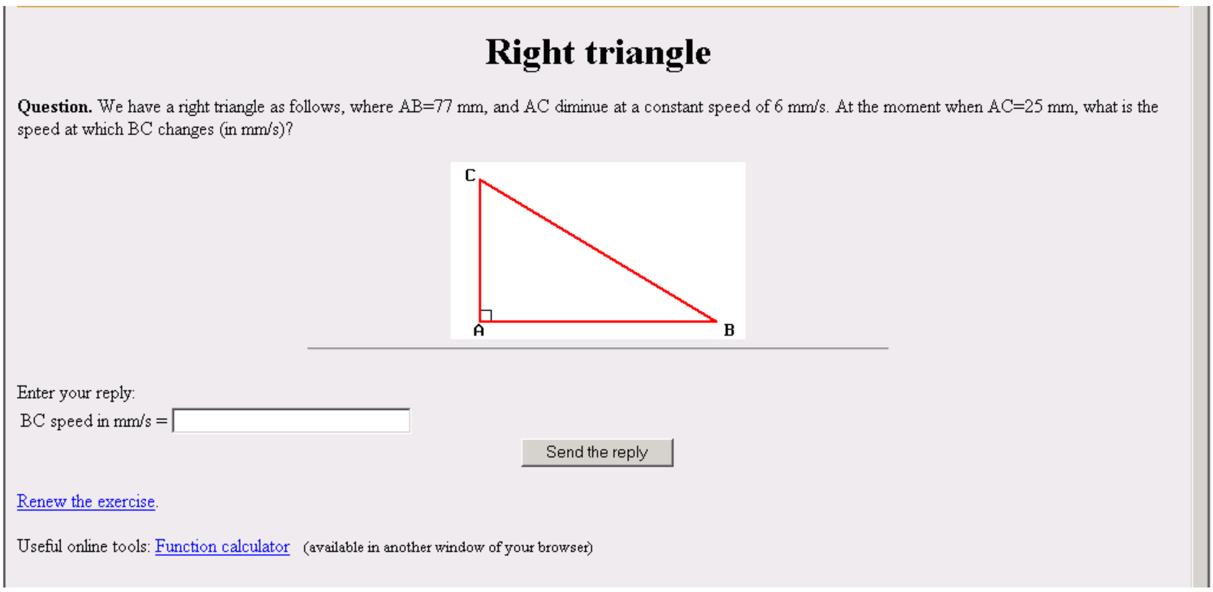
In Braise, several kinds of helps are available for that exercise[3]:
- short courses, recalling general results that can be useful for that exercise;
- description of general methods, that must be transferred by the student to that particular case;
- a graphic help, displaying of the first terms of the sequence;
- hints for each of the two questions.

All these helps are proposed simultaneously; none of them reduces the students' activity to a mere application of properties or routines. However, a detailed solution of the exercise is also available. Some of the students could fake looking for a personal solution, and only try to understand the solution proposed by the computer. The study of the log files will show if it really happens.

Wims

The exercise we focus on (exercise W) requires a mathematical modelling of a geometrical situation. It takes randomly three forms: "right triangle", "circle", or "tower". Figure 2 displays a screenshot of Wims, with the exercise in its "right triangle" form.

Figure 2: Screenshot of Wims- Exercise W



There is no help proposed in Wims for that exercise[4]. The students have access to a functional calculator. Even if the answer awaited is numerical, that exercise requires

---

[3] *English version of the text:*

Let (u_n) be the sequence defined as follows: u_0 is given, $u_0 \geq -2$, and for all n: $u_{n+1} = \sqrt{2+un}$

We want here to compare to elementary methods to study that sequence.

a)According to u_0 study if the variation of the sequence. Deduce that (u_n) is convergent.

b) Use the definition of convergence to study, according to u_0, the convergence of (u_n). State the rate of convergence.

many personal ideas, choices and decisions of the student. He or she has in particular to adapt his or her reasoning to the three possible forms of the text. Thus that exercise is, like exercise B, likely to lead students to a real problem-solving activity.

III. Students' behaviour

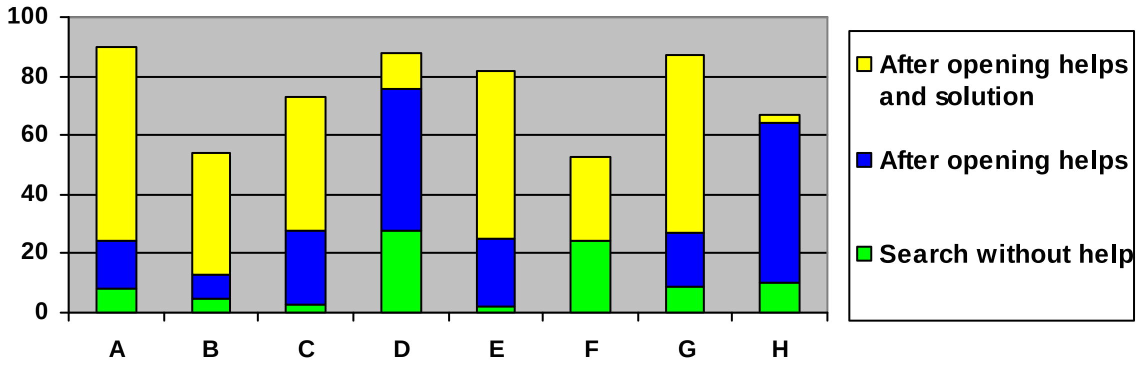
For each experiment, the software provides log files that allow us to follow the students' activity in detail. We demonstrated in the preceding section that the chosen exercises can lead the students to a real problem solving activity. We will now use the log files to examine precisely the students' behaviours, in order to determine if they really develop such an activity, and in particular if they do not use the computer to fake producing a personal solution.

Braise – Exercise B

The log files of the eight pairs of students working on Braise provide the connection time on each possible window (hints, courses, graphic help, solution…). The first thing we observe is a real involvement of the students in the task. The average time spent on the exercise is 1 hour and 13 minutes. In a traditional tutorial session in France, the time spent on such an exercise rarely exceeds half an hour (after that time the teacher usually proposes a solution[5]).

However, the students working on the computer look at the helps, and at the solution, after a quite short time. We present the corresponding numerical values on figure 3. It displays three stages of the students' work: a search without any help; after opening the "helps" window, but not the solution; and after opening the "solution" window. The times are indicated in minutes on the vertical axis.

Figure 3: Synthesis of the students' activity with Braise.



We observe on that graph a well-known result about the work in mathematics with a computer: the students can work at their own pace. It leads them to very different attitudes. More precisely here, we observe a prevailing attitude shared by the groups A, B, C, E and G. They open the "helps" window during the first nine minutes, and

[4] A similar exercise is proposed with a graphing calculator at the Dutch National examination 2002 (Drijvers 2004)

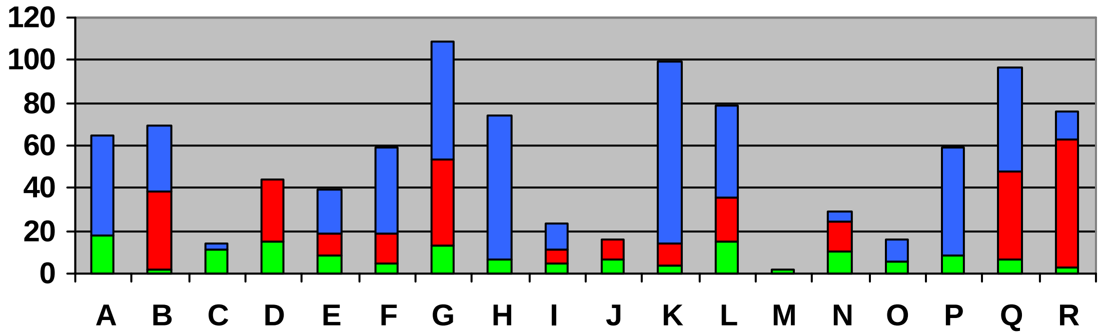
[5] The observations about traditional tutorials stem from a master's dissertation: Sylvie Le Merdy (2003), "Problem-solving at the university and in preparatory classes".

the “solution” window within the 25 following minutes. B, C and E even look at the helps during the first five minutes. It means that they open the corresponding window right after reading the text of the exercise, before any personal attempt to solve it. Some of these students could certainly find at least a partial solution without any hint. For all these students, more than a half of the time is spent after displaying the solution. The more extreme case is B: the students look at the solution after only 13 minutes, just after reading the helps.
However, the time spent after opening the solution’s window is not only dedicated to reading and understanding it. A precise study of the log files shows that all the pairs who opened quickly the solution’s window closed it at one moment. Even if they open it again later on, for these students (A, B, C, E, G), the time spent with the solution’s window open represents almost 60% of the third stage of their work. It indicates that they use the solution as a very detailed hint. According to the direct observation, it seems that they read it quickly and then try to produce a similar reasoning on their own. It is not a problem solving activity in a strict sense, but it is a real personal work of the students.
Besides, all the students wrote questions and remarks for the teacher on their logbook[6]. And they succeeded in solving a similar exercise at home.

Wims - Exercise W

The figure 4 displays for the 18 pairs:
- The time spent before the first answer (*lowest zone*);
- The additional time needed to reach the maximum mark of 10; it can be zero when the first proposition of the student is correct (*middle zone*);
- The additional time, where the students go on working on the exercise after reaching the maximum mark, which can also be zero when the students do not work after reaching the mark 10 (*upper zone*).

Figure 4: Synthesis of the students’ activity with Wims.



The numerical values confirm again the real involvement of the students (an average of 54 minutes work on the exercise, including 16 minutes of personal work outside of

[6] For the sake of brevity, we will not present here examples of these remarks. They will be presented in the complete paper.

the organised sessions). They also show a great variety of attitudes between the different pairs of students: the total time spent on the exercise ranges from 2 min (M) to 1h 49 min (G).
The first stage, before making a first attempt of answer, is relatively short: an average of 8 min. All the students reach the maximum mark at one moment, but the time spent on that second stage changes a lot. There is no help from the computer for that exercise, and very often the teacher intervened to provide hints. He sometimes even indicated the solution. That is the reason why everyone reached the mark 10. However, after reaching it most of the pairs went on working on the exercise by themselves during a long time. Some of these were not able to reach the mark 10 again.
Besides, many pairs renew often the exercise in order to obtain their favourite version of the text (triangle, circle or tower). Perhaps some of these guess the right answer for one of the configurations without having the slightest idea of the mathematics results involved.
The final paper and pencil exam comprised the “triangle” and the “circle” version of that exercise, and a new variation of it, with a sphere. The results obtained by the students prove that most of them really understood the exercise: over 36 students, 24 succeed with the triangle and the circle, and 20 of these also succeed with the sphere.

IV. Conclusion

Let us go back to our initial question: do these softwares allow the students to develop a real problem solving activity? For a first kind of students, the ones who search by themselves during a long time before proposing a solution, the answer is clearly positive. But they represent only a minority of the students we observed. For all the others, the answer is not obvious. They undoubtedly develop a real mathematical activity, spending a long time working with the solution (Braise) or working on the exercice after reaching the maximal mark of 10 (Wims). They were able afterwards to solve similar problems, so they clearly learned mathematics. Can we claim that they developed a problem solving activity, even if they worked with the solution’s window open, or with a solution provided by the teacher? One can answer positively, because all these students needed at one stage to produce a personal solution, adapted from the model. But that question must be discussed. It indicates the need for further studies, especially in order to produce more precise descriptions of the students’ activity on one exercise after looking at the solution or at least at a correct answer.
Anyway, we observed in both experiments that the students adopted very different behaviours. It goes further than the usual observations about the possibility of work at their personal pace with a computer. The observations exposed in part III prove that very different working patterns are developed. Some students spend a long time looking for a solution by themselves (Braise) or before proposing a first answer (Wims). On the opposite, others ask very quickly for the solution (Braise) or renew often the exercise (Wims). Moreover, that flexibility in the activities choice and

organisation at the exercises' scale has also been observed at the scale of a sequence. Is that flexibility a reason for the greater involvement of the students? Does it help the learning and in which way? And how can the teacher cope with that relative freedom of the students? These questions are mentioned in several research works about the use of computers. We intended to contribute to their study in further research using in particular the grids presented in part II.

References

Castela C. (2000) Un objet de savoir spécifique en jeu dans la résolution de problèmes : le fonctionnement mathématique. *Recherches en didactique des mathématiques 20(3), 331-380.*

Cazes, C., Gueudet, G., Hersant, M., Vandebrouck, F. (2004) Using web-based learning environments in teaching and learning advanced mathematics, *ICME 2004*

Crowe D., Zand H. (2000) Computers and undergraduate mathematics 3: Internet resources, *Computers & Education, 35 (2), 123-147.*

Drijvers, P. (2004) The integration of technology in secondary mathematics education : future trend or utopia ? *ICME 2004.*

Hu O., Trigano P., Crozat S. (2001) Une aide à l'évaluation des logiciels multimédias de formation in E. Delozanne, P. Jacobini (ed.) *Interaction homme machine pour la formation et l'apprentissage humain*, numéro spécial de la Revue Sciences et Technologies éducatives vol 8 n°3-4.

Robert A. (1998) Outils d'analyse des contenus mathématiques à enseigner au lycée et à l'université, *Recherches en didactique des mathématiques 18(2),139-190.*

Robert A., Rogalski M. (2002) Comment peuvent varier les activités mathématiques des élèves sur des exercices ? Le double travail de l'enseignant sur les énoncés et sur la gestion en classe, *Petit x, 60, 6-25.*

Ruthven, K., Henessy S. (2002) A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning, *Educational studies in mathematics, 49 2-3, 47-86.*

Schoenfeld, A. H. (1985) *Mathematical problem solving.* Orlando, Academic Press.