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Methodological tools for Comparison of learning theories in technology enhanced learning in mathematics

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Summary

This deliverable summarizes the collaborative work developed by TELMA teams on theoretical frames, and presents a methodological tool for systematic exploration of the role played by theories in technology enhanced learning in mathematics.

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Rationale of this Document

This document reports on the collaborative work developed by TELMA teams on theoretical frames during the two first years of existence of Kaleidoscope. This collaborative work began by an identification of the main theoretical frames used by TELMA teams in their research and a first attempt at understanding how these theoretical frames influence the visions they develop as regard technology enhanced learning in mathematics. This first phase of the collaborative work was mainly based on the descriptions provided by the teams and the analysis of some of their most representative articles. In a second phase, in order to deepen the reflection and to better understand the exact role played by the theoretical frames they use in their research, TELMA teams decided to organize a cross-experimentation where each team would experiment an ICT tool it had not produced, and to organize their collaborative work around this cross-experimentation. It was also decided to built a methodological tool for the systematic exploration of the theoretical frames used in technology enhanced learning in mathematics, which would support the analysis of the cross-experimentation, and also would aim, beyond this particular experimentation, at supporting the understanding of the role played by theoretical frames in the design and analysis of uses of ICT tools and the search for interesting connections and complementarities between such frames.

This document presents the different steps of this collaborative work, its main outcomes up to now, and how TELMA teams plan to continue their work in the next year.

TELMA Teams

(ITD) Consiglio Nazionale Ricerche – Instituto Tecnologie Didattiche – Genova - Italy

(UNILON) University of London - Institute of Education – London - UK

(DIDIREM) University Paris 7 Denis Diderot – DIDIREM - Paris - France

(ETL) National Kapodistrian University of Athens - Educational Technology Lab – Athens - Greece

(MeTAH) MeTAH and Leibniz – IMAG – Grenoble - France

(Siena) University of Siena - Department of Mathematics – Siena – Italy

1. Introduction

Since the beginning of their collaborative work in the Kaleidoscope network of excellence, the different teams involved in TELMA have been struck by the diversity of the theoretical frames they used. A better mutual understanding of these theoretical frames, of the exact role they played in their respective work on technology enhanced learning in mathematics, the search for connections and complementarities between these, emerged thus as a necessity for developing an effective collaboration.

As a first step in this direction, it was decided that each team would prepare a synthetic description of the main theoretical frames it used, and would send to the other teams a reduced set of articles it considered especially insightful for understanding the type of research the team developed, the theoretical frames it relied on, and the way these influenced its work both in the areas of design and use of ICT tools. About 40 papers were thus selected. Most of these are posted on the TELMA website[i] and can be downloaded from it. This first phase of the work led to an internal report entitled « Theoretical frameworks of reference » (December 2004) which is also accessible on the TELMA website. This report presents a synthetic description of the theoretical frameworks used by the different teams, points out some problematic issues of common interest and tries to figure out how each of the theories tends to frame the ways these problematic issues are addressed. Three issues are considered: the notion learning environment, the relationships between teacher and learner in a teaching and learning process, and the role of instruments in teaching and learning processes. The report also introduces the notion of « dida[1]ctical functionalities of ICT tools », proposing to use it for clarifying the role played by theoretical frames in the work of the TELMA teams. This notion has been further elaborated in a contribution offered at CERME4 by three Italian researchers from TELMA (Cerulli & al., 2005).

This first step resulted in a quite useful improved communication between the different teams, and led to interesting conjectures about the ways theoretical frames shaped their respective vision and work on technology enhanced learning in mathematics (see part 2 below). Nevertheless, it soon appeared that it presented evident limitations due to the characteristics of the corpus used: a selection of published papers. In such papers, most often, the theoretical frames used are presented in a synthetic way or even just referenced in a specific part at the beginning of the text, but it is difficult to infer from what is written the exact role these frames have played in the research work carried out. This difficulty has been already pointed out in a meta-study carried out by DIDIREM (Lagrange & al., 2003), and also discussed in a specific Working Group on theoretical frames at CERME 4 (Dreyfus & al., 2005). Better understanding this role, beyond a pure declarative level, is nevertheless necessary in order to determine what are the exact consequences of different choices of theoretical frameworks and what are the exact needs in terms of connections and complementarities.

Understanding the role played by theoretical frameworks is all the more difficult as, if one adopts a cultural vision of research[ii] (Hall, 1959), theoretical frames influence the research work not only explicitly at a 'technical' level, but also implicitly at 'formal' and 'informal' levels. In order to understand this role, one has thus to enter more in the intimacy of the research work, what the reading of published papers hardly allows. The same limitations were experienced in the other facets of the TELMA work on representations and contexts. Thus the decision taken at the end of 2004 to prepare a cross-experimentation where each team would experiment an ICT tool it had not produced, and to organize the TELMA work about theoretical frames, representations and contexts around this cross-experimentation. The constraint imposed to the teams: experimenting an ICT tool they had not produced was seen as a way of fostering deeper exchanges between them. It was also expected that the distance thus introduced between the designers and the users would create specific didactic phenomena and make visible effects of theoretical frames and contexts not visible in the first corpus.

It was also decided to give a privileged role to PhD students and young researchers in this cross-experimentation, in coherence with the expectations expressed by Kaleidoscope managers of having the sub-structures of this network contributing to the Virtual Doctoral School and using the on-line discussion facilities offered by its platform. This choice had as a counterpart that the cross-experimentation was reduced to a few number of sessions as PhD students and young researchers could not be overloaded with extra work.

Starting from documents sent by the teams responsible for the TELMA collaborative work on theoretical frames, representations and contexts, the group of PhD students and young researchers involved in the micro cross-experimentation prepared first a guideline for this experimentation (cf. Annex 1). Each team also chose a particular ICT tool, analysed it using its own theoretical frames and prepared an experimentation to be carried out at the beginning of the academic year 2005-2006. During that time, the other

TELMA researchers continued their reflection on theoretical frames, representations and contexts, and as regard theoretical frames, they engaged in the building of a methodological tool adapted to a systematic investigation of the role played by theoretical frames in development and research about technological uses. This methodological tool, while being developed in an autonomous way and with the aim of designing a tool for a more general use, benefited from the first phases of the cross-experimentation process.

In this report, we first synthetize the results of the first phase of the TELMA work on theoretical frames evoked above, then we present the way the micro cross-experimentation dealt with theoretical issues, before introducing the methodological tool which has been designed and will be tested and refined in 2006. We end this report by some comments on the work already achieved and the plans for year 3. Complementary information on the micro cross-experimentation can be found in the deliverables on representations and contexts and on the TELMA website.

2. Some characteristics of the theoretical frameworks used by TELMA teams

As has been explained above, the first step of TELMA work on theoretical frameworks resulted in an internal report presenting the different frames, introducing the notion of didactic functionalities of an ICT tool, and trying to figure out how the theoretical frames they used influenced the design and research work of the TELMA teams.

We present here some parts of this report that we find especially insightful for understanding this first phase of TELMA work, and then make some general comments.

2.1 The report on "Theoretical frameworks of reference"

The report presents first in a synthetic way the main aims of the research developed by each team and its theoretical frameworks of reference. These are summarized in the table below.

Team	Main research aims	Theoretical frameworks of reference
DIDIREM	Understanding the dialectics between conceptual and technical work, instrumental genesis processes, taking into account the institutional dimension of learning processes Developing ICT tools for algebra and functions	Theory of didactic situations (*Brousseau*) Anthropological didactic theory (*Chevallard*) Instrumental approach *(Rabardel, Artigue, Lagrange, Trouche)*
ETL	Understanding the generation of mathematical meanings in ICT environments, and the influence on these of classroom norms Understanding the role of the teacher in the classroom and the changes introduced by ICT tools	Situated abstraction (*Noss & Hoyles*) Classroom norms (*Cobb & Yackel*) Socio-constructivist approaches (*Lerman)* constructionism and deep structural access to technologies *(Papert's group, Di Sessa)*
ITD	Studying how new technologies can contribute to the construction of innovative environments that can enhance learning processes and change traditional approaches to teaching	Activity theory (*Engeström*) Microworlds and related theories (*Papert*, *Di Sessa*) Situated abstraction (*Noss & Hoyles*)
MeTAH	Design and development of Aplusix learning environment for algebra, and analysis of its use, students' modelling in algebra	Theory of didactic situations *(Brousseau)* CKC (Balacheff) Artificial intelligence concepts (*Anderson*)
Siena	Understanding how meanings rooted in activities involving artefacts can evolve towards mathematical meanings under the guidance of the teacher by means of peculiar	Semiotic mediation (Vygotsky) Activity theory (*Engeström*) Microworlds and related theories (*Papert*, *Di Sessa*) Theory of instruments

	semiotic practices, and how artefacts can be better exploited as instruments of semiotic mediation	(*Rabardel*) Theory of instruments of semiotic mediation (*Mariotti*)
UNILON	Studying the nature of mathematics and mathematical activity as it is constructed in a text, what relationships do the author and the reader have to each other and to the subject matter, and the role which the text plays within a particular situation	Social semiotic perspective *(Halliday)*

Table 1 : Reference theoretical frameworks for TELMA Teams

The report then points out three problematic issues of common interest for the different teams and elaborate on these, comparing how they are approached through different theoretical lens. These problematic issues are the followings: the notion of learning environment, relationships between teacher and learner in the learning process, the role of instruments in teaching and learning processes.

2.1.1 The notion of learning environment:

As regard this notion, the report first points out its evolution in the last decades as a result of the technological development and its role in educational practices, and also as a consequence of the theoretical frames evolution. From a vision of learning as an individual process whereby knowledge emerges from the interaction between the student and the computer, and a conception of the learning environment reduced to the software itself, one has moved in recent years towards a vision of learning as a social process and a conception of the learning environment as something including the whole teaching and learning situation, considering the whole set of interactions established in a class over the course of time and how the activities evolve. All TELMA teams share this 'holistic' vision of the notion of learning environment, even when they refer to different frameworks in order to define and interpret it. The report then examine how two of the main approaches are used by TELMA teams in order to grasp this social dimension of learning processes: the theory of didactic situations on the one hand and activity theory on the other hand, frame the notion of learning environment. We reproduce below this part of the report that contrasts the two approaches by opposing the cooperative vision underlying the notion of learning environment in activity theory and the antagonist vision underlying the same notion in the theory of didactic situations.

"TDS: learning environment as 'milieu' antagonist of the subject

Referring to the Piaget's theory, Brousseau says that student learns by means of adaptations to the 'milieu' which is source of contradictions, difficulties and disequilibria; but a 'milieu' without didactical intentions is not sufficient for the student to acquire the knowledge the teacher would like him/her to acquire.

An important part of the work of devising didactical engineering in Didactical Situations Theory is to find a fundamental situation for teaching a mathematical concept (a-didactic situation), which will be the point of departure to create an antagonistic system for pupils. Bound by the didactical contract, pupils know they have to behave in a given situation by acting on it. Acting creates retro- actions and from this dialectical process pupils' knowledge is born. So teaching, in this theory, needs this antagonistic system, which is named the 'milieu'.In the TDS, learning environment is seen as system that is antagonist to the learner. The 'milieu' opposes retroactions to the answers or to the inadequate choices of the student with respect to the a-didactic situation presented. In order to learn, a student has to understand as insufficient his/her control of the situation. The 'milieu' is not an allied of the student but it is a competitor. In fact: "si l'enseignant cherche à organiser un milieu *allié* où l'acteur agit sous des contraintes *qui essayent de lui faire éviter les confrontations*, alors nous sommes en face *d'interactions de type fictif* Dans l'apprentissage par adaptation, il s'agit au contraire de construire des connaissances contre un milieu antagoniste qui résiste. En effet, ce sont les rétroactions du milieu qui permettent l'apprentissage de l'élève. Dans un milieu allié, il n'y a pas de rétroaction, l'élève agit, le milieu " est agit " (Margolinas, 2001)

In order to better describe how the notion of 'milieu' (learning environment) is presented in TDS, it is necessary to put in evidence the role of the teacher and the role of artifacts in teaching/ learning situations. One of the roles of the teacher is to construct the conditions under which the responsibility of the solution of the task is entirely submitted to the student. This process is named 'devolution'. Between the moment in which student accepts the problem as his/her own problem (and not as a school problem) and the moment in which he /she produces the solution, the teacher has to step aside: the student has to construct his/her knowing. Another role of teacher concerns the institutionalisation of the acquired knowledge. The artefacts contribute to structure the material 'milieu' in which student acts. The interaction with artefacts, in fact, gives students retroactions which allow students to develop new strategies of solution.

<u>AT: Learning environment as cooperative activity oriented to an educational goal</u>

In the AT frame, the learning environment is constituted by the enactment of a teaching/learning activity oriented to an educational object, involving student, teachers and artifacts. Studying the learning environment means studying the teaching/learning activity oriented to a didactical objective.

Moreover, the cooperative and social character of a human activity has been highlighted in the Vygotskian frame. This aspect brings a cooperative connotation to the notion of learning environment. A learning environment is something which is negotiated, co-built in the teaching and learning activity by participant of the activity, and it evolves during the development of the activity. Thus, it is not something which is assigned and constructed a priori.

AT provides a model to describe the structure of any human activity, and its transformations occurring along with its evolution. It is the model proposed by Engeström and Cole, which can be used also to describe the system of relationships characterizing a teaching/learning activity, and thus to describe a learning environment.

This model assigns a crucial mediation role to the instruments, the rules, and the division of labour in the three relationships characterizing any human activity that is the relationships between subject and object, between subject and community, between community and object.

By means of this model it is possible to describe the nature of the cooperation that characterizes the activity and that is indispensable for achieving the object.

According to this model, the teacher is a co-actor of the activity, and artefacts are instruments that mediate the subject's action, the subject-community communication, and the variety of roles, duties and obligations characterising the relationship between community and object.

The model clarifies the mediations and the relations that determine the potentialities of fostering learning, highlighting its social nature.

Moreover the model is used to highlight the evolution that an activity can undergo during its development when contradictions or breakdowns occur, forcing a change of focus in the activity, thus forcing a transformation of its structure.

To sum up, the main difference between learning environments in TDS and AT, is that: in the TSD the learning environment is the 'milieu' that is antagonist to the subject; whilst in AT, the learning environment is the cooperative activity, oriented to an educational aim. »

This presentation can certainly be discussed, and the assimilation of the notion of learning environment to the antagonist milieu associated to a-didactic situations considered as a reduced vision of what can be offered by the TDS for approaching the notion of learning environment. But it has the merit to point out interesting differences between the ways these two theoretical frames tend to frame the approach of the social dimension of learning processes, central to each of them.

2.1.2 Relationships between teacher and learner in a teaching-learning process

As regard this issue, the report complements the comparison of TDS and TA by considering socio-constructivist approaches such as those developed by Cobb and Yackel (Cobb & Yackel, 1996). The excerpt of the report quoted below points out the role played by specific notions for conceptualizing the sharing of mathematics responsibilities between teachers and students: the didactical contract in the theory of didactic situations, division of labour in activity theory, and socio-mathematical norms in socio-constructivism.

"Theory of Didactic Situations: the didactical contract

As previously remarked, the theory of didactic situations (TDS) is influenced by Piaget's theory, but, as stated by Brousseau, in Piaget's theory the teacher may be discharged of any didactical responsibility: a milieu without didactical intentions is not sufficient for the learner acquire all the knowledge the teacher would like him/her to acquire. As a consequence the role of the teacher is thus fundamental in the TDS, so are the relationship between teacher and learner within an ongoing teaching and learning process. Such relationships, in TDS, are called *didactical contract*.

The didactical contract is not a generic pedagogical contract, because it depends strictly on the corpus of knowledge on which a given teaching and learning process focuses. It is a set of relationships that determines what are the responsibilities of the teacher, and of the learner, with respect to each other. Some of such relationships can be explicitly stated, but most of them are implicit. It is this system of reciprocal obligations that can be defined as being a contract; what characterizes such contract as being didactic is its part specifying educational contents: the aimed mathematical knowledge.

The reciprocal obligations of the contract cannot be enunciated, because of their strongly implicit nature. What results to be particularly important is the *breakdown of the contract* (ex. a student's surprise when asked by the teacher to accomplish a task he/she is not able to accomplish; the teacher's surprise when he/she thought that the student was able to accomplish the task, and that his/her explanation was sufficient, etc.). In fact, also the rules of the contract, like any other form of learning, are interiorized through a process of assimilation and accommodation. Constructing of a didactical contract for the learning of given knowledge takes place through a dynamic process in which contradictions / breakdown may emerge. These appear as breakdowns between what the teacher expects in terms of the student's acceptance of obligations and the load of responsibility that the student's is able to bear when tackling tasks.

Overcoming these breakdowns can lead to adaptation phenomena that do not bring about effective knowledge within the class (for example the Topaze effect, the Jourdain effect etc.). On the other hand, the breakdown may be overcome through the search for a new contract based on the readjustment of the previously.

Activity theory: division of labour

As previously stated, the influence of Vygotskian theory on the Activity Theory determines the cooperative and social character of the activity as the engine of the learning process.

Within this context, the relationships between teacher and student can be interpreted as mediators between community and object of the activity. The teacher is part of the community because participates to the activity sharing the same object that the student (subject) has to learn. The relationship between community and object is called *division of labour*. The division of labour refers to the explicit and implicit organization of community as related to the transformation process of the object into the outcome.

According to the activity theory model, belonging to a community implies a division of labour, that is, the repeated and negotiated distribution of work tasks, power, and responsibilities among the participants. In practice, the division of labour defines a system of reciprocal obligations that mediate the strategy by which community members, interpreting specific roles, interrelate for the social construction of the object of the activity.

The division of labour is built along with the development of the activity; it can evolve by means of breakdowns forcing a change in the focus of the activity. For instance, the teacher may realize that he/she charged the student of a responsibility that he/she is not able to manage. It may then emerge a contradiction (the student is not able to accomplish a task) that changes the focus of the activity: from 'accomplishing the task' to 'providing the students with the elements needed to be able to accomplish the task'. This leads the teacher to revise his/her role in the activity assuming, for instance, a more cooperative role and consequently modifying the division of labour.

Socio-constructivism: socio-mathematical norms

The notion of socio-mathematical norms can be contextualized in the socio-constructivist framework. The idea is that students construct their knowledge and understanding of the world not just through direct personal experience and discovery (constructivist paradigm), but also through the intellectual sharing and support of those around them (socio-constructivist paradigm). In this perspective the teacher plays a crucial role.

The teacher is perceived in terms of her/his role in organizing the class, setting up tasks for the students to be engaged in and supporting their learning process. Within this context, the relationship between teacher and student, with respect to the learning of a given mathematical concept, is defined in terms of *sociomathematical norms*.

These norms are distinct from general classroom social norms in that they are specific to the mathematical aspects of student' activity. For example, the understanding that students are expected to explain their solutions and they ways of thinking is a social norms whereas the understanding of what counts as an acceptable mathematical explanation is a socio-mathematical norms.

In general, we may say that the concept of *sociomathematical norms* is comparable with the *didactic contract* of the TDS and with the *division of labour* in the Activity Theory. Indeed they all define the role of the subjects (teacher and students) that are involved in a teaching and learning process with respect to a mathematical concept.

2.1.3 The role of instruments in teaching and learning processes

As regard this point, the report first explains that, when talking about an instrument, it will consider: an agent (or subject), an instrument[iii] (a concrete or non concrete object), an objective that the agent tries to achieve by means of the instrument, and how the given instrument can be a means for achieving the given objective. Then the report points out that an ICT tool in an educational practice must be considered as an instrument at two clearly different levels:

- the level of an educational instrument where the agent is the teacher and the objective is the specific teacher's educational goal,
- the level of a practical instrument where the agent can be any user, and especially here the student.

Then the role of instruments is compared taking into account three different theoretical frames, once more the theory of didactic situations and activity theory, and Rabardel's theory. For the first two frames, one can see evident connections with the previous analysis in terms of learning environments, as shown by the following excerpt:

"The role of instruments for the theory of didactic situations

When an instrument is employed in school practice, as we stated, we may analyse both, the practical and the educational level.

When a student is using an instrument to solve a problem, we say that he/she using the instrument at the practical level. Within this framework, the learning outcomes resulting from the use of an instrument at the practical level are discussed in terms the interaction of the learner with the *milieu antagoniste*. If an instrument is a component of the *milieu*, and if a learner is an agent using the instrument for a practical objective, then we can interpret the interaction between agent and instrument in terms of the interaction of the learner with the milieu; if that is the case, then we may consider the learning outcomes as being the result of the adaptation of the learner to the milieu in consequence to the retroactions of the milieu on the learner himself/herself. Thus, if an educator wants to employ an instrument at the *educational level* he/she has to set up situations in which the instrument is part of the milieu and is employed by the learner as a mean to accomplish the proposed task. The nature of the learning outcomes is discussed in terms of situation that is being set up, and in terms of the nature of the milieu, thus it is particularly important to study the nature of the employed instrument (ICT tool in our case), and its retroactions on the user, according to the chosen educational goal.

<u>The role of instruments for the activity theory</u>

In the case of the Activity Theory, an instrument is considered to be mediating both the relationship between subject and objective of an activity, and the relationship between subject and community in the sense that the introduction of an instrument may change the rules of the community and the division of labour. Within this theory, the learning environment is not considered as antagonist to the subjects (as in the case of the *milieu antagoniste* of the didactic situations theory); on the contrary, it is considered to be a cooperative environment. The interaction between a learner and the learning environment is interpreted in terms of cooperation, and not in terms of opposition, in this sense, the *agent* using an instrument to achieve an *objective*, is considered to be cooperating with the instrument as being part of the learning environment. When a learner uses an instrument at the practical level for achieving an objective within an activity, the learning outcomes are considered to be structured by the nature of the activity itself and by role played by all its components. Consequently, within this theory, in order to employ an instrument as an *educational instrument* for achieving a given educational goal, an educator has to set up an activity in which the instrument mediates the relationship between the learner and an objective (at the practical level) that is relevant for the given educational goal. Moreover, for the activity to be effective with respect to the chosen educational goal, the educator has to consider how the employed instrument structures the activity and the relationships among the different components of the activity itself. In other words, the educational functionalities of the instrument are defined in terms of how the instrument may structure an activity, rather then on the retroactions given to the user as in the case of didactic situation theory; of course also such retroactions are to be considered, because they influence the relation between learner and instrument, but they are not they main focus.

The role of instruments for the Rabardel's theory

According to this theory, when a subject uses an instrument to accomplish a given kind of task, he/she passes through the process of instrumentation that results in the internalisation of the schemes of use (Rabardel, 1995) of the instrument in the form of techniques that can be applied to a whole class of tasks that are somehow compatible with the given kind of task. In other words, when an instrument is used at the *practical level*, the process of instrumentation results in a learning outcome consisting on the internalization of certain techniques associated to the schemes of use of the instrument, and that can be applied to a wide class of tasks.

Suppose that an educator aims at a given mathematical educational goal which in particular involves pupils to learn how to accomplish certain tasks, or to learn certain mathematical techniques, and then it is possible to employ a suitable instrument as an *educational instrument*. The educator may choose an instrument to be used by the learners in order to accomplish certain tasks that are relevant to the given educational goal. Then the instrumentation process may lead the learner to internalize the schemes of use of the instrument in the form of techniques that are coherent to the aimed mathematical ones, and that can be applied to the mathematical tasks individuate by the educational aim.

Within this framework, the didactical functionalities of an ICT tool are strongly dependant on its schemes of use that structure the learning outcomes derived from the employment of the instrument at the *practical level*. As a consequence, in order to employ an instrument as an *educational instrument* a special attention has to be put on the schemes of use of the instrument when it is used to accomplish a given kind of tasks"

2.2 Comments

This first phase of the TELMA work evidences the diversity of the theoretical frames used by TELMA teams, but also some common trends, which transcend this diversity. The first one is without any doubt a common sensitiveness to the social and cultural dimensions of learning processes. As has been pointed out, this common sensitiveness reflects a general evolution observed both in the domain of mathematics education research and in the domain of EIAH research. As regard mathematics education, synthetic visions of learning epistemologies such as that provided in (Sierpinska & Lerman, 1996) point out the progressive evolution from pure constructivist approaches of learning processes towards socio-constructivist and socio-cultural approaches, and the resulting shift in dominant references from Piaget to Vygotsky for instance. As regard EIAH research, a similar evolution has been observed as attested for instance in (Pernin, 2005). Analyzing the successive visions of the functions to be integrated in technological artefacts for learning, this author identifies five successive approaches respectively focused on behaviour, knowledge, exploration and discovery, learning objects, activities and situations. He also points out the increasing importance taken by the last one, and how it is supported by the technological evolution itself. In TELMA, according to the particular research culture of the different teams, the sensitiveness towards social and cultural dimensions is supported by different constructs, from those borrowed from activity theory or social semiotics to constructs elaborated inside the field of mathematics education itself such as those related to the theory of didactic situations and the anthropological approach.

Quoting Tchounikine (Tchounikine, 2004), Pernin also stresses the necessity of maintaining the quality of didactic reflection which tends to disappear from some recent models. The tendency is to focus on the complex architecture of technological artefacts and on their potentialities for enhanced interactions inside networks of actors. Then, the characteristics of the knowledge implemented in these artefacts and the potential content of interactions can be overlooked.

This is not at all the case for the TELMA teams. Both in the design and the use of ICT tools, they share a common sensitiveness to the ways mathematical objects are implemented into ICT tools and to the possible cognitive and didactical consequences of this implementation. As a consequence of this sensitiveness, they reject the common vision of technology as a simple pedagogical adjuvant and share the conviction that ICT tools deeply affect mathematical learning both in its forms and contents. Concepts such as semiotic mediation, computer transposition of knowledge and instrumental genesis support this sensitiveness.

Two characteristics of the theoretical positions adopted by TELMA teams were evidenced by the first phase of the collaborative work : (1) a theoretical diversity well representative of the current general trends in mathematics education, (2) shared visions and common sensitiveness on some crucial points. This confirmed our conviction that studying how our theoretical diversity support these shared visions and common sensitiveness, could be both an accessible and productive enterprise. We also expected that connections and complementarities emerging from this study, could interest researchers beyond the sole TELMA teams.

At the same time, as mentioned in the introduction, we were nevertheless aware of the methodological limitation of the work developed so far. It did not allow us to understand clearly what was taken in charge by theories in the design or analysis of use, what was not and why, or to understand how precisely theoretical frames shaped the decisions taken. It was insufficient thus for understanding the exact consequences of our theoretical choices, for identifying the precise needs we had in terms of connections, and interesting complementarities. Hence a methodological choice was done: organizing our exchanges and collaborative work around a common project of cross-experimentation where each team experiments a tool designed by another team, and to organize the experimentation and its analysis around a unifying notion , the didactical functionality of an ICT tool.

2.3 The notion of didactical functionality

The notion of didactical functionality of an ICT tool was defined in the report mentioned above in the following terms:

"Given an ICT tool, it is possible to identify its didactical functionalities: with didactical functionalities we mean these properties (or characteristics) of a given ICT, and their modalities of employment, which may favor or enhance teaching/learning processes according to a specific educational aim".

"The three key elements of the definition of the didactical functionality of an ICT tool are then:

1. a set of features / characteristics of the tool
2. an educational aim
3. modalities of employing the tool in a teaching/learning process referred to the chosen educational aim."

This notion was presented as a means for contrasting the use of ICT tools for educational purposes and for other purposes (in the latter case, no didactical functionality is taken into account), and also professional and educational ICT tools. As explained in (Cerulli & al., 2005):

"An educational ICT tool provides, because of its nature, a set of such functionalities. In fact we assume that the producers of the tool, not only design it with respect to a set of specific educational goals, but we assume that they also consider the possible modalities of employment of the tools in order to achieve such goals. In other words educational ICT tools are designed with a set of didactical functionalities. Nevertheless professional ICT tools may provide features that can be interpreted in terms of didactical functionalities, that is, we can identify modalities of employment of such tools aiming at the achievement of a given educational goal [...] Thus in the case of professional ICT, the definition of didactical functionalities occurs only in the utilization phase, whilst in the case of educational ICT, they surely occur in the design phase, but may also occur in the educational use phase."

Beyond this contrasting role, at the light of the reflections developed so far, we consider this notion as an interesting tool for anchoring our theoretical reflection in the real tasks that one has to solve when designing or analysing effective uses of ICT tools. It is thus a methodological tool for taking some distance from the relationship to theoretical frames involved in the first step of the collaborative work, helping us to move, in terms of theory, from the declarative to the operative.

3. The organization of the micro cross-experimentation

In this part of the report, after recalling its aims, we describe how the micro cross-experimentation has been organized and data collected with regard to the role played by theoretical frames. We then synthesize the first data collected from the different teams.

3.1 Aims and organization

The micro cross-experimentation is still in progress. What is expected is to better understand the ways theoretical frames influence the analysis of a given tool and of the potential it offers for mathematics learning. We also aim to better understand how this potential is exploited in a particular context - that is to say the vision one has of its didactic functionalities in a particular context – and how the results of this exploitation are presented and analysed.

This has to be achieved:

- by a reflective analysis to be made by PHD students and young researchers involved in this experimentation,
- by a search for similarities and differences between this reflective analysis and the analysis, choices and anticipations which have supported the design of the ICT tool and the didactic functionalities the designers had in mind,

- o and, finally, through the search for possible reasons for these similarities and differences.

For that purpose, we decided to design a guideline questionnaire in order to collect the necessary information without putting too much or too difficult work on the shoulders of the young researchers and PhD students involved, and also something understandable by all the researchers involved in the experimentation, whatever their didactic and technological culture.

The guideline questionnaire is thus an object of compromise. As mentioned in the introduction, each team responsible of a dimension of the collaborative TELMA work (that is to say theoretical frames, representations and contexts) produced a proposal submitted then to the group. From these three proposals, the group built the guideline questionnaire selecting questions for composing a global questionnaire.

The cross-experimentation is a micro-experimentation. This characteristic introduces evident constraints in the choice of the ICT tool and in the didactic exploitation made of its learning potential. There is no doubt that the scenarios built and the results obtained will not reflect comprehensively the affordance of the selected ICT tools. They will neither reflect comprehensively the affordances of the different theoretical frames as we can hypothesize that some components of these will not be really pertinent in this particular context.

This has of course to be taken into account in the ways this experimentation and its results will be used, but we also want to stress that we make the hypothesis that analysing the choices made by the different teams in this highly constrained context will be especially interesting as regards the questions at stake, by revealing how the constraints are perceived and how they influence the choices implicitly or explicitly made.

3.2 The first proposal of guidelines and the rationale for it

We reproduce below the initial document proposed to the PHD students and young researchers involved as regard theoretical frames.

Proposal of guidelines for the joint experimentation : Theoretical frames

For preparing this proposal, we started from the point that the theoretical frames we rely on shape our research work in different ways we are more or less conscious of, more or less able to articulate. Two types of questions are thus proposed: the first ones just ask for precisions about the way the experimentation is designed and then analysed without specific reference to theoretical frames. The second ones ask explicitly about the theoretical frames used and the ways the team thinks that these influence its work. Both types of questions are supposed to give insight in the ways theoretical frames shape the joint experimentation and its outcomes.

A second point is that theoretical frames will influence the experimentation through the identification of didactical functionalities for the selected tool, the design of the experiment, the choices made as regard the data collected and the analysis carried out, the interpretation made of these, and the results obtained. Thus the questions address these different moments of the experimentation process.

A third point is that we see this experimentation as a way of questioning what we perceive, after one year of reflection:

- as similarities between our approaches and concerns such as the sensitiveness to the social and semiotic dimensions of mathematical activity and learning processes, and the sensitiveness to instrumentation processes,
- as potential integrative notions such as the notion of didactic functionality.

What are exactly the similarities and differences in the ways we approach these common concerns? If we think in terms of integration, what could be reasonable priorities? Is the notion of didactical functionality a notion easy to make operational and a useful one? We hope that the experimentation will offer us insights on these issues and this, of course, reflects in the proposal.

The last point is that questions have to make sense whatever be the culture of the team and answered by young researchers in a reasonable time. Thus we have tried to avoid formulations too tightly linked to a particular culture, and limited the number of questions.

I. General questions

Designing the experimentation:

- QG1 : What are the precise aims of your experiment and the questions you want to focus on?
- QG2 : Why did you choose this product and why do you think it is appropriate for approaching these questions?
- QG3 : What kind of analysis of the software do you think important to develop, and what tools do you use for that?
- QG4 : What do you see as the main didactical functionalities of the software? How did you determine these, and how did you choose the particular functionality(ies) you want to study?
- QG5 : What are your main choices for the conception of the sessions? How do you see the role(s) of the teacher and the student-teacher interaction in these?
- QG6 : How do you take into account in this experimentation the social dimension of learning processes? What is already offered in your opinion by the software itself in terms of social interaction, and how do you complement it in the experiment?
- QG7 : How do you take into account in this short term experiment the necessary familiarisation of the teacher and the students with the software (instrumentalisation process)? How does this influence the way you use the software?
- QG8 : Which data do you plan to collect and which analyses do you plan to carry out with these?

After the experimentation:

- QG9 : What were the changes introduced with respect to your previsions in the practical realisation and what were their reasons?
- QG10 : What are, in your opinion, the main outcomes of this experimentation?
- QG11 : What questions does it raise?

II. The theoretical choices and their influence

- QT1 : What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?
- QT2 : In your opinion, in which ways do your theoretical choices have influenced:
  - the analysis of the software and the identification of its didactical functionalities (software features, educational aims, modalities of employement including the configuration of the software)?
  - the conception of the experiment?
  - the choices of the data and their analysis?
  - the results you obtain and the conclusions you draw from these?

3.3 The final guideline questionnaire and its possible exploitation

We present here the final guideline questionnaire resulting from discussions in the group on the above document. A priori questions and a posteriori questions are distinguished with respect to the implementation of the experimentation. There are 7 a priori questions, 5 a posteriori questions, three of these being asked twice; it is then asked if there is any difference in the answers given in the two phases of the work.

QT1 and QT2 have been included in the guideline questionnaire, and among the general questions, only QG1. In fact, PHD students and young researchers decided to focus on questions more precisely related to the three dimensions : theoretical frames, representations and contexts. Finally we have the following list :

A priori questions	QG1, QT1, QC1, QC2, QC3, QR1, QR2
A posteriori questions	QT2, QC3, QR1, QR2, QR3

Table 2 : Guideline questions for the cross-experimentation

The Qc (questions on context) and Qr (questions on representations) are the following:

QC1 : What is the type of research that you follow (eg. Classroom based, case studies) and how is this related to the kind of your research focus ?

QC2 :Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way ?

QC3 : How do you capture / analyse the role of the tools in pupils problem solving processes or solutions ?

QR1 : What forms of feedback are provided ? How are solutions validated and by whom (e.g. by the tool itself, by the teacher, by peer or self-validation) ?

QR2 : What is the « distance » between the objects and the means of manipulating provided by the tool and those used in paper and pencil based work within the target domain ?

QR3 : Do users also use other modes of representation not provided by the tool itself (e.g. paper and pencil representations, calculator) ? What are these and what does their function appear to be ? How do these modes of representation relate to those provided by the tool ?

While focusing on representations and contexts, the answers to these questions are also of course of interest for the analysis to be carried out on theoretical frames, and they will be used in order to compensate the fact that the general questions of the first proposal have not been inserted into the final guideline. More precisely, QC2 can thus be put in relation with QG4, QC3 with QG8, QR1 with QG4, QG5, QG6, QR2 and QR3 with QG3. Only two general questions are more difficult to link to questions in the guideline: QG2 asking for the reasons behind the choice of a particular tool among those a priori available, and QG7 related to the restrictions induced by the characteristics of this experimentation.

3.4 The answers of the different teams to the a priori question concerning the theoretical frames used

We synthesize in the table below the answers given by the different groups to the question asking what theoretical frames they were using in the cross-experimentation.

Team	ICT Tool used	Theoretical frames used
DIDIREM	Ari-Lab	Ergonomic and Instrumental approaches - Theory of didactic situations – Anthropological approach
ETL	Ari-Lab	Constructionism and socio-cultural approaches - Situated abstraction – Semiotic mediation – Instrumental approach
ITD	Aplusix	Socio-constructivist approaches
MeTAH	Ari-Lab	Instrumental approach - Anthropological approach
Siena	Aplusix	Vygostkian theory concerning internalisation of control
UNILON	E-Slate	socio-cultural and social semiotic approaches

Table 3: Theoretical frames used in the cross-experimentation

These choices are coherent with the theoretical positions of the TELMA teams identified above in part 2. The motivations given for supporting these choices confirm the common sensitiveness these teams have for the social and cultural dimensions of learning processes, the attention they pay to the students' role in the construction of knowledge, the prominent role they give to semiotic and instrumental approaches in order to analyse the learning potential of ICT tools and actualize this potential in the cross-experimentation. This is for instance attested by the following excerpts of their answers.

ETL:

"This choice of our team is motivated by the estimation that they [the theoretical frames choosen] seem to be bringing into the foreground some fundamental issues in the process of learning and teaching: a) the role of the social setting where the learning activity is integrated b) the acknowledgement of the student's role in the knowledge construction c) the investigation of the role of tools and representations in the process of learning and teaching."

UNILON:

"The focus of our research is shaped by a socio-cultural and social semiotic theoretical framework. This assumes that the meanings available to participants in a particular activity or setting are structured by the semiotic resources available and by the contexts of situation and of culture.
In studying teachers' and students' use of the E-Slate tool in the classroom we are interested to see how the new semiotic resources available to the classroom participants are taken up and coordinated with other, more familiar, means of representation. Specifically, we have been looking at the ways in which the 'slider' representation of fraction is employed through physical manipulation and through oral and written means of communication."

ITD

"We adopt the socio-constructivist paradigm: we assume that Aplusix can be used to set up and devlop open ended and problem solving activities which will foster pupils construction of problem solving strategies. Pupils will work in pairs or groups, and this will help them in constructing and make explicit their strategies in relation to the activities. However, such new strategies may be or may be not coherent with the teacher's educational goal, but such a coherency can be achieved by means of the interaction of the teacher with the single groups or by means of institutionalization activities (still driven by the teacher) with the class as a whole."

Siena

"We adopt the Vygotskian theory in order to study how the control offered by Aplusix can influence the behaviour of the students towards errors and impasse. According to this theory, we formulate the hypothesis that the feedbacks provided in the microworld determines a change in the attitude towards errors and impasse."

MeTAH

"1. Artifact / instrument

Our goal is to study the effects of using a computer-based tool (ARI-LAB2) on the learning of the concept of fraction. Within this theoretical framework, we are interested in studying instrumental genesis in pupils working with the fraction microworld of ARI-LAB2 software.

2. Anthropological theory (concept of praxeology)

Our purpose is first, to investigate the types of tasks that can be given and that are meaningful in the fraction microworld of ARI-LAB2 software, and, second, to search for tasks and techniques that allow developing an appropriate instrumental genesis for fractions."

But, as can be noticed, the information given by these answers remains rather vague for one who wants to really understand how these theoretical choices have impacted the identification of didactic functionalities and the practical decisions taken for organizing the experimentation. This is the reason why in the DIDIREM team we decided to complement these answers by an "entretien d'explicitation" with the PhD students and young researchers involved in the experimentation (see part 4 below).

4. A methodological tool for systematic exploration of the use of theoretical frames

As mentioned in the introduction, the definition of the methodological tool we present in this part of the report has been partially independent from the organization of the cross-experimentation. Nevertheless, after some preliminary comments, we use data coming from the preparation of the cross-experimentation in order to make more understandable the kind of phenomena this methodological tool is expected to help identify and analyze hoping that this will help the reader to understand the choices made in its elaboration. We then present a first version of the tool structured around the notions of didactical functionality and concern.

4.1 Preliminary comments

These preliminary comments explicit some of the choices that supported the elaboration of the methodological tool. They result from a reflective analysis of the work developed so far as regard theoretical frames within TELMA.

1. The different theoretical frames used by the TELMA teams support their research work on technology enhanced learning in mathematics, enlightening some important dimensions while other ones remain into the shade. Our conviction is that the first needs to be satisfied in the work undertaken are those concerning areas where different lights focus on, such convergence being of course tightly linked to the common sensitiveness we have recalled in part 3. Thus the methodological tool has to be designed in order to address this common sensitiveness while respecting the existing diversity in the approach of these.

2. We also consider that what we need a methodological tool allowing us to better understand what makes the specific coherence of each different vision, and what are the consequences of this coherence in terms of strengths and weaknesses of the corresponding frame. It is assumed that such a tool will help to establish productive links between different frames, and support partial integrative views when these appear accessible and possibly productive, keeping in mind that a global integration is certainly out of reach, and even not desirable, the strength of any approach being attached also to the specific lens it chooses for approaching the complexity of the reality we study.

3. We have chosen to structure the methodological tool around the idea of didactical functionality. As has been stressed in part 3, this choice is seen as a means for approaching the real functioning of theoretical frames, putting these in relation with effective decisions taken as regard the use of technological tools, and trying to go beyond a declarative relationship to theoretical frames. We thus question the theoretical frames through the way they shape explicitly but also implicitly the vision of didactical functionalities, the means used for identifying and exploiting these, and the ways one retrospectively looks at such exploitations. We make the hypothesis that any theoretical choice conditions the vision of didactical functionalities, through the three components attached to this notion, enlightening only some facets of these.

4. We have also decided to associate to each component of the notion of didactical functionality, a set of « concerns », expressed in the most neutral way, for identifying the respective areas of light and shade. Then the analysis will try to determine for each of these concerns (1) if it is addressed or not, (2) the respective importance given to it if addressed, (3) the associated problematization, (4) the language used and concepts mobilized, (5) the theoretical frames these expressions can be more or less directly related to, and of course, (6) the effect of these on practical decisions taken in terms of design or analysis of the educational use of ICT tools.

In order to make more understandable what can be expected from such a methodological tool, we present now a particular example: the preparation of the micro cross-experimentation by the DIDIREM team.

4.2 One particular example

The DIDIREM team has chosen for this experimentation the tool Ari-Lab produced by the team ITD-CNR. We first present how the team identified different didactic functionalities and selected between these. We then try to clarify the exact role theoretical frames have played in this identification and in the preparation of the experimentation. This is an a posteriori reconstruction where, using a methodology of «Entretien d'explicitation » (Vermersch & Maurel, 1997), the team tried to go beyond what was explicitly expressed by the group of PhD students and young researchers in their answers to the guideline questionnaire. The assumption was that the informal level of their shared didactic culture had influenced, often unconsciously, their reflection and the decisions taken as regard the experimentation. The «Entretien d'explicitation » aimed the emergence of their understanding and uses of theoretical frames at this informal level.

We report this in a synthetic way, just in order to help understand the kind of work, our methodological tool has to be able to support. This of course does not substitute to the analysis of their experimentation with Ari-Lab which will be produced by the PhD students and young researchers of DIDIREM.

4.2.1 The identification of didactical functionalities

The information provided by the designers of Ari-Lab present this tool as a set of inter-connected microworlds. This was confirmed by the first inspection of Ari-Lab by the team . As in any microworld, some abstract concepts are reified into the microworld and embodied action on these abstract concepts is accessible through the direct manipulation of their representations. The interface looks attractive and its design seems especially well adapted to the elementary school context in France, even in the first grades. Among the diverse microworlds, for reasons of institutional compatibility, the numerical microworlds have been judged the most adequate for such a short experimentation. That is why the numerical microworlds and the context of elementary school were chosen.

With Ari-Lab, it is possible to work in the numerical domain in different ways, which allows a variation both in contexts and systems of representations. This also contributes to open the space of accessible strategies to solve a given problem, according to the microworld which is used and the interaction between microworld and paper and pencil techniques. Moreover a specific emphasis has been put in this tool on the development of interaction capabilities both for collaboration between students and interaction between teachers and students.

These characteristics of the tool will suggest two ways for the didactic exploitation of the tool in the group: one giving the priority to the interaction between microworlds in problem solving, the other to the interaction between students for the collective elaboration of solutions of a complex problem and a reflexive work on these solutions.

Because of the short time allocated to the experimentation (3 sessions), the team had to select one of these possibilities.

The group more deeply explored the different numerical microworlds searching for those a priori best adapted to his experimentation. This implied a deeper look at the characteristics of the respective « milieux » associated to the different microworlds : implemented objects and associated representations, possible actions on these objects and associated feedbacks, and of course some evaluation of the distance between these objects and representations and those familiar to the pupils and used in French schools at this level. This analysis was not made by using a specific grid of inspection or evaluation, but through personal then collective exploration and rather informal discussions.

In spite of its shortness, the experimentation has also to support the general goals of mathematics teaching at this time of the academic year. For these grade 2 pupils, what is at stake at this time is the extension of the field of numbers towards numbers greater than 100 and the preparation of the algorithm for subtraction through the development of personal techniques. With the help of the teacher, pupils have to explicit the role played by the decomposition of numbers, in the effectiveness of these techniques. The new mathematics syllabus for elementary school asks teachers not to limit to the canonical decompositions using the base 10 numeration and to encourage diversity in the use of decompositions, in order to prepare the automatization of calculation without penalizing the flexibility which is necessary to mental calculation or to what it calls « calcul réflechi ». The syllabus indeed emphasizes the importance of mental and reflexive calculations, and also the necessary progression from personal techniques to standard techniques in the teaching of arithmetic operations.

The selection of microworlds was the result of a progressive elimination. Due to the small number of sessions, due to the age of the pupils, familiarization with the selected microworlds had to be quick and easy. Problems of cultural or institutional compatibility had to be systematically avoided (for instance, in the number micro world, a comma separates the groups of three digits instead of a space in the French usual representation of whole numbers, the comma being used for separating the integer and decimal part of numbers, role played by a dot in Ari-Lab; in another micro world, operations are posed in a way unusual in France).

Moreover, this analysis of the different microworlds reveals some ergonomic problems (in terms of position and navigation in the micro world, of interpretation of icons, of number of actions to be performed for a given elementary task). The group wanted to limit as far as possible the possible effect of these characteristics on pupils' activity during the experimentation. Finally two microworlds were selected: « Money » and « Abacus ».

Two modalities of use were a priori envisaged as mentioned above, trying to benefit from two different didactic potentialities.

The first one uses only one microworld: the microworld « Money » and focuses on the potential offered for collaborative work between pupils, for solving an open problem based on number decompositions. The group thought of a classical problem such as the following: « Find the greater possible number of different ways to get a given amount of money with given banknotes and coins ». Starting from this generic task, a progression can be built, by playing on supposed didactic variables such as the size of the target number, the types and the numbers of banknotes and coins available. A devolution of the task can be achieved by using a small target number and a few types of coins or banknotes, and then some « saut informationnel » can be introduced. The microworld favours decompositions through the exchange possibility and the validity of these exchanges is automatically checked.

Collaboration between students can make it possible to find all the solutions in a reasonable amount of time, and motivate discussions on the different strategies used by the pupils. In a later phase, constraints could be added or removed, and ways could be looked for economically finding the new set of solutions. Such new problems could lead to a move from solutions based on action in the microworld towards solutions integrating mental calculations, and also motivate an evolution of the semiotic representations used in order to make them more effective, and finally launch a reflexive work on the semiotic representations themselves.

The second modality was based on the joint use of two microworlds: « Money » and « Abacus », and focused on the elaboration of personal techniques involving decomposition of numbers for calculating differences between numbers which are out of the field of calculations already met by these pupils. The microworld « Money » was supposed to favour the diversification of strategies and a flexible use of decompositions while the microworld « Abacus » was expected to support the use of canonical decompositions.

After discussion, the group chose this second modality. They postulated that it would be easier to implement and to negotiate with the teacher as it seemed less distant from her own practices and closer to her current goals.

This choice led to a scenario of use structured around three different phases with a small group of pupils (6), including a phase of familiarization with the two microworlds, then a succession of tasks where the pupils would freely choose the microworlds they want to use and the strategies for solving the proposed tasks.

The two first phases were themselves structured into three moments:

- a first encounter with the microworld (a collective phase),
- solving the task, the pupils working in pairs,

- o a collective discussion on the strategies used and their results, the difficulties met, and a local institutionnalization with both a mathematics and a technological dimension.

The collective encounter was organized, and then the task proposed to the pupils was selected in order to favour an a-didactic functioning in the second moment and the production of a set of experiences and results rich enough to nourish the collective discussion under the responsibility of the teacher.

In the third phase of the experimentation, an a-didactic functioning mode was expected, but a particular attention would be paid to the devolution, to be sure that the students understand correctly the rules of this new game, and the autonomy given to them in the choice of the microworlds, in the articulation between paper and pencil work and work in the microworlds, in the choice of calculation and representation modes. It was planned to organize the collective discussion at the end of this session around the strategies used, their more or less adequacy to the different calculations involved in the solving of the given tasks, the role played by decomposition of numbers and the ways these express according to the systems of representation used.

We will not enter into more details in the description of the preparation of this experimentation. This level of description seems to us sufficient for our purpose, which is the following: identifying the theoretical frames implicitly or explicitly used by the group and investigating the exact role they have played.

4.2.2 The theoretical frames used and their roles in the preparation:

We think that three different theoretical frames at least have been more or less consciously used by the group, in the identification of didacticalfunctionalities before the experimentation itself, but here a posteriori reconstructed : the instrumental approach, the theory of situations and the anthropological approach. A background of epistemological and didactic knowledge as regard the numerical conceptual field had also a complementary but crucial influence.

The table below displays the main roles played by these four factors, in relationship with the three dimensions identified in the notion of didactical functionality.

DF/TF	Instrumental Approach	Theory of Didactic Situations	Anthropological Theory	Epistemological and Didactic Knowledge on numbers
Characteristics of the tool	x	x	x	x
Educational goals			x	x
Modalities of use	x	x		

Table 4 : Theoretical frames and the determination of didactical functionalities

More precisely:

The Instrumental Approach

In this instrumental approach, we include the ergonomic analysis that the team had to make from the first inspection of Ari-Lab. This kind of analysis entered into the culture of DIDIREM thanks to the participation of members of this team to the multidisciplinary project AIDA[iv], and thus to research work developed in that area. It has been then used in different research projects of DIDIREM about on-line resources for mathematics learning, and especially in a regional project carried out at high school level (Artigue & al., 2004). This use obliged the team to consider the tools and criteria used in ergonomics for analysing educational multimedia tools, and adapt these to its specific contexts of use and needs. The inspection of Ari-Lab, from an ergonomic perspective, mainly concerned its « usability ». Issues related to « acceptability »[v] could also have been taken in charge by this ergonomic analysis but the group preferred for that purpose to refer to the concepts of the anthropological approach, which they were more familiar with. The ergonomic analysis led to the elimination of microworlds, supported the choice of the components of the selected microworld that would be officially introduced, the way the familiarization with the two microworlds was planned, and the terminology used in that phase.

The fundamental concepts of the instrumental approach: instrumentalization, instrumentation and instrumental genesis were used in a different way. The notion of instrumentalisation together with the notion of utilisability mentioned above supported the organization of the first encounter with the two microworlds. The definition of the tasks proposed to the pupils also took advantage of this notion: identifying how pupils had to instrumentalise AriLab to solve the projected tasks, choosing the task variables, planning their management while keeping in mind that the instrumentalization needs had to be as small as possible.

The instrumental approach was also engaged in anticipating the knowledge to institutionalise, especially knowledge related to the tool itself and the instrumented strategies and techniques. Moreover, being aware of the fact that the duration of an instrumental genesis is necessarily longer than this micro-experimentation, from the start, the group tried to have moderate and realistic ambitions in that respect. This also made it sensitive to the fact that, in such a short experiment, it could not really study the potential for mathematics learning of a tool as complex as Ari-Lab, but only some very limited facets of this potential.

The Theory of Didactic Situations

The tools of the instrumental approach and above all, those of the ergonomic analysis of ressources were rather new didactic tools for the PhD students and the young researchers, and this characteristic made their use most often conscious; this was not at all the case for the theory of situations they had met and used from their first encounter with the didactic research field, and which was really part of their didactic culture. Identifying up to what point this theory was engaged in the reflections they developed and in the decisions they took, is thus much more difficult.

The retrospective analysis we have carried out shows that it has been used in the analysis of Ari-Lab and in the selection of the two microworlds. It supported the particular attention paid by the group to the possibilities of action offered to the pupils, to the nature of the feedback possibly received, and the fact that these characteristics were interpreted using the notion of « milieu ». Some microworlds for instance were eliminated because the system of feedback they proposed was too much limited as compared with what is generally expected from a « milieu » offering a-didactic potential for learning. This also induced to distinguish between feedbacks consisting in just a validation of pupils' answers and feedbacks more elaborated, and more likely to support pupils' strategies evolution, and mathematics knowledge development.

Together with epistemological and didactic knowledge in the conceptual field of numbers, in the precise design of the modalities of use, both in the global organization of the didactic scenario, and the detailed elaboration of each session, specific attention was paid to the devolution and institutionnalization processes, to the definition of types of tasks, then of particular tasks generated by fixing didactic variables, to the organization of the mathematical progression through the evolution of these didactic variables. This was classically performed through an a priori analysis where the group tried to evaluate the a-didactic potential of its scenario, and from this evaluation, the possible sharing of mathematical responsibilities between the pupils and the teacher in the different phases of the work, and what the teacher could do to enrich the «milieu», if necessary.

The Anthropological Theory

The team used the anthropological theory to take in charge the institutionnal analysis mainly in the first two dimensions of the notion of didactical functionality. In our opinion, the strong importance given by the group to issues of didactic legitimacy and institutionnal distance in their reflection and in the design of the experimentation attests the role played by this theory in their approach of the experimentation, although the group did not explicitly use the notion of praxeology, which is at the core of the theory. Note that the notion of praxeology could certainly have been used in the design of the tasks proposed to the pupils if the use of the tools provided by the theory of situations had not been something so strongly attached to engineering design in the culture of the team.

In the first phase of the work, some microworlds were eliminated for reasons of institutional distance. For instance, at first, the group found quite interesting a microworld on fractions based on the representation of rational numbers on the real line. The underlying mathematics refer to the Thalès theorem (as it is called in France). Several interesting types of tasks could be designed for this microworld, compatible with the curricular expectations at the end of middle school. But the group thought that it would be very difficult to negociate these with middle school teachers at that moment of the academic year because these would like to be able to have their students mobilize the Thalès theorem in order to understand the mathematics underlying the techniques used in the microworld, in other terms in order to be able to develop the kind of « technological discourse » – with the meaning given to this term in the anthropological theory - they think they would have to provide students with or help students develop by themselves. Classically, the Thalès theorem is taught later in the academic year and they estimated that it was not realistic to ask a teacher to substantially change its mathematics organization of the academic year, just to be involved in a micro-experimentation. The group thus moved to the idea of experimenting at elementary level using numerical microworlds, and even there, as has been explained above, some microworlds were discarded because of institutional distance.

The anthropological approach was also used for choosing a didactic goal, making the group especially sensitive to the necessary compatibility of this choice with the didactic goals of the teacher at that moment of the academic year. The group was also sensitive to the professional overload that could result for this elementary teacher, volunteer but with limited relationship to research, from an experimental design too much distant from her usual practices. We hypothesize that these concerns, although not articulated as such, influenced the decision of the group when they had to choose between the two modalities of use initially thought of. We have attached this last sensitiveness to the anthropological approach but, in this precise case, due to the culture of the group, it would certainly be more reasonable to link it to concerns inheritated from the theoretical culture the group has built about research on teacher practices, and especially to the « double approach » developed by Robert and Rogalski (Robert, 2001).

As shown by this retrospective analysis, the theoretical frames mentioned above have played a decisive role in the preparation of this experimentation. They have been explicitly used but, as it had been hypothesized, their intervention was also very often implicit, the theoretical frames acting as « informal » elements of the didactic culture of the group. Due to the particular status of this experimentation and its organization, it seems sensible to conjecture that the balance between the explicit and implicit has moved in favour of an explicit use, but also that the retrospective analysis was nevertheless not able to reveal all the « naturalized » interventions of the theoretical frames, especially because this retrospective analysis was undertaken within the same didactic culture. Structuring the analysis around the notion of didactical functionality was helpful, by focusing the attention on the different dimensions involved in the design of this experimentation and by drawing the attention towards the associated choices and decisions, leading us to investigate the possible rationale for these. The general impression resulting from this analysis is that what is explicitly controlled by the theory in the identification of didactical functionalities is like the proverbial iceberg's emerged part. To improve the communication between teams of researchers, we obviously need to access also substantial elements of the immerged part.

The work carried out on this particular example was inspired by the reflection developed so far on theoretical frames and what was needed in terms of methodological tool. It worked also as a test upon our preliminary choices and contributed to the design of the methodological tool we present in the next section.

4.3 A methodological tool for the systematic exploration of the role played by theoretical frames in the use of ICT tools

To structure this methodological tool, we use as announced above the notion of didactical functionality. In each of its three dimensions, the methodological tool provides a set of what we call « concerns ». In different frameworks, not all of these concerns are considered or given the same emphasis, and even when they seem to be given a similar importance, they are not necessarily expressed, dealt with in the same way, with the same conceptual tools, and the decisions taken can diverge. This is through the identification of the respective attention given to these different concerns, and the precise ways they are approached that we try to elucidate the role played both explicitly and implicitly by theoretical frameworks, to identify interesting connections and complementarities, and also divergences, potential misunderstandings and conflicts we need to be aware of.

As stressed above, the theoretical frames we are the most familiar with very often function in a naturalized way. Understanding their exact role requires thus a real work of retrospective reconstruction that the methodological tool is expected to support efficiently. From this point of view, the diversity of theoretical perspectives offered by the different TELMA teams is especially helpful. What is naturalized in the work of one particular team will not necessarily be naturalized in the work of the other ones.

We have thus selected for each of the three dimensions of the notion of didactical functionality, a set of concerns we consider potentially informative, taking into account what we know about the theoretical frames used by TELMA teams and current research in the area of technology enhanced learning in mathematics. Each of these concerns is given more or less importance in the theoretical frames used by TELMA teams, and the common concerns are generally approached in different ways. When expressing these concerns, we have tried to find formulations the most neutral as possible, not too much attached to a particular theoretical frame.

4.3.1 Tool analysis and identification of specific tool characteristics :

The analysis of a tool associated to the definition of didactical functionalities generally involves two different dimensions, questioning on the one hand how the mathematical knowledge of the domain is implemented in the tool, and on the other hand the forms of didactic interaction provided by the tool. Both the implementation of the knowledge of the domain and the didactic interaction can be approached through different perspectives, which are not independent, neither mutually exclusive. The analysis and decisions resulting from the choice of such or such perspective are, among other factors, dependent on the theoretical frames referred to and on the ways these are used.

We have selected according to this dimension the height followings different concerns :

- concerns regarding tool ergonomy (TE)
- concerns regarding the characteristics of the implementation of mathematical objects and of the relationships between these objects (IMO),
- concerns regarding the possible actions on these objects (AMO)
- concerns regarding semiotic representations (SR)
- concerns regarding the characteristics of the possible interaction between student and mathematical knowledge (ISK)
- concerns regarding the characteristics of the possible interaction with other agents[vi] (IA)

- concerns regarding the support provided to the professional work of the teacher (teacher support : TS) - concerns regarding institutional and/or cultural distances (ICD)

Table 5: Concerns regarding tool characteristics

4.3.2 Educational goals and associated potential of the tool

It is more the relationship between potentialities and goals rather than each of these considered separately which can contribute to enlighten the role played by theoretical frames according to this dimension, complementing what is offered by the information provided by the analysis of the tool. We hypothesize that the choices made and the ways these are expressed and justified, partially reflect the theoretical frames explicitly or implicitly mobilized or the general principles underlying these.

Considering the theoretical frames used in TELMA, it seems to us interesting to investigate the relative importance given in the definition of educative goals to considerations of epistemological nature referring to mathematics as a domain of knowledge or as a field of practice, considerations of a cognitive nature focusing on the student in her relationship with mathematical knowledge, considerations focusing on the social dimension of learning processes, and finally institutional considerations.

Thus the concerns we selected for this dimension are the four followings:

- Epistemological concerns focusing on specific mathematical contents or specific mathematical practices (E) - Cognitive concerns focusing on specific cognitive processes, or specific cognitive diffficulties (C)

- Social concerns focusing on the social construction of knowledge, on collaborative work (S) - Institutional concerns focusing on institutional expectations, or on the compatibility with the forms and contents valued by the educational institution (I)

Table 6: Concerns regarding educational goals

4.3.3 Modalities of use

The design of modalities of use and the a priori analysis of their implementation supposes a multiplicity of choices of diverse nature. It is reasonable to hypothesize that only a small part of these are under the control of theoretical frames, explicitly or even implicitly, many other being dictated consciously or unconsciously by the educational culture and the particular context within the realization takes place.

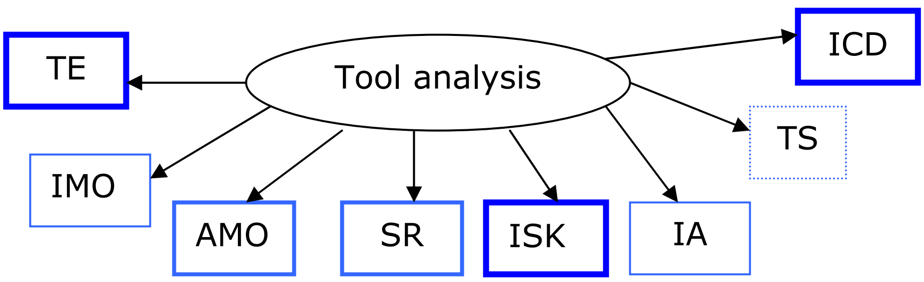
What we find significant is to identify what is given a privileged role among this potential diversity, and the ways the choices made are expressed. The categories of concerns we propose to include are here the 7 followings:

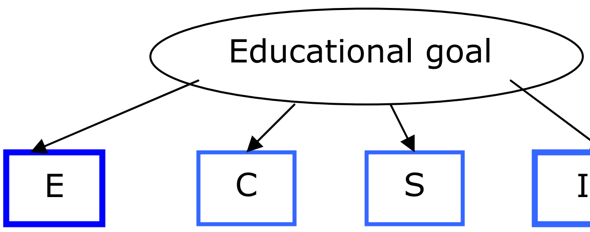
- Concerns regarding contextual characteristics (CO) - Concerns regarding the tasks proposed to the students including their temporal organization and progression (TA) - Concerns regarding the functions given to the tool including the possible evolution of these (TF) - Concerns regarding instrumental issues and instrumental genesis (IG)

-	Concerns regarding the social organization, and especially the interactions between the different actors, their respective roles and responsibilities (SO)
-	Concerns regarding the interaction with paper and pencil work (PP)
-	Concerns regarding institutional issues and especially the relationships with curricular expectations, values and norms, the distance with usual environments (ID)

Table 7: Concerns regarding modalities of use

What has been defined here is a multidimensional structure that we consider as a tool for questioning and analysing educational uses of ICT tools. Coming back to the particular example of the AriLab experimentation we presented above, one can see that, for each of the dimensions, some particular concerns are given a high priority while other do not seem to play such a leading role although they are also taken into consideration. We have tried to visualize these below by introducing four levels of emphasis:





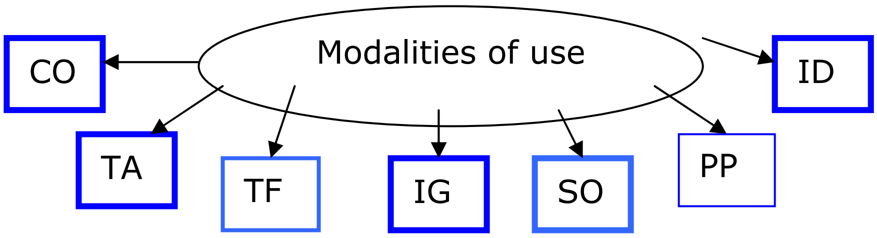


Figure 1 : Distribution of concerns in the definition of didactic functionalities (DIDIREM Team)

We can hypothesize that this distribution of concerns is influenced by the theoretical frames used by the researchers, and the detailed analysis we have carried out help us understand the underlying mechanisms. The detailed analysis shows a more complex picture than the above visualization, the three levels being inter-related and the leading concerns at each level influencing the ways the other ones are managed. The structure will certainly be different for the other teams as well as the underlying mechanisms and we hope that the use of this methodological tool will contribute to make the comparison insightful.

There is nevertheless no doubt that theoretical frames, although they influence how didactical functionalities are perceived and exploited, only very partially condition the effective use of ICT tools. In the analysis that this methodological tool will support, it is thus important to be careful and not overestimate the real influence of theoretical frames.

5. Conclusions and previsions for future work

In this report, we have tried to show the evolution of the collaborative work undertaken in TELMA about theoretical frames, from the identification of the different theoretical frames used by TELMA teams and the analysis of a selection of published papers to the organization of the micro cross-experimentation and the design of a methodological tool for the systematic exploration of the role played by theoretical frames in the design and analysis of uses of ICT tools.

This is an on-going work, and for instance the methodological tool presented above has just been used up to now for analysing the role played by theoretical frames in the definition of didactical functionalities on a particular example. We need to test it with the constructions made by the other TELMA teams and investigate its potential for supporting the comparison of their respective constructions as well as the development of connections and complementarities between our respective theoretical frames. For that purpose, we need to invest it not only in this kind of a priori analysis but also in the retrospective analysis of the cross-experimentation. Theoretical frames will certainly play also an essential role in this retrospective phase, different from the role played in the a priori phase, and as insightful as the a priori use as regard the questions we want to address through this cross-experimentation.

Another point deserving discussion is the notion of didactical functionality itself. Separating three levels helps us to structure the reflection, and also the methodological tool. But the first analysis carried out clearly shows that the three levels are neither independent nor chronologically ordered. The analysis of an ICT tool is influenced by the conjectures and anticipations one makes as regard its didactic potential and modalities of use. This interaction was for instance evident in the preparation work of the DIDIREM team. How to adequately take into account these interactions in the use of the methodological tool is a question to be addressed in our future work.

This future work will be organized in the following way:

1. Test by each team of the methodological tool in the a priori and a posteriori analysis of its experimentation, starting from the documents prepared by the groups of PhD students and young researchers of the team. We think that the kind of methodology based on "entretiens d'explicitation" used by DIDIREM can be helpful here.
2. Feed back from the teams having produced the different technological tools used in the cross-experimentation. How do they react to the visions of the didactical functionalities identified and tested through the cross-experimentation? What is the relationship between these and what they tried to incorporate in the ICT tool?
3. Collective work on these different analysis, looking for possible and interesting connections and complementarities, pointing out also perspectives less easily reconciliable, clarifying possible misunderstandings.
4. Revision of the methodological tool in the light of the different analysis carried out, enriching it if necessary, and integrating comments and illustrative examples taken from the experimentation for enlightening the meaning of the concern analysis and its possible affordances.

We also would like to point out that what this collaborative work aims is the understanding of the role played by theoretical frames in the design or analysis of uses of ICT tools, not in the design of such tools. This is another task beyond the scope of TELMA. Although our construction can be helpful for that purpose, other categories are certainly necessary in order to take into account the different forms of theoretical knowledge involved in the design of ICT tools and the ways these influence the decisions taken in the design process. One can also hypothesize that in design, it is a more global vision of didactical functionalities which is at stake as compared with the one used here.

6. References:

Artigue, M. (2002). Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7, 245-274.

Artigue, M. (Ed.) (2004). *Suivi et évaluation d'un projet d'expérimentation de ressources en ligne en mathématiques en classe de seconde mené à l'initiative de la région Ile de France*. http://pcbdirem.math.jussieu.fr/SITEscore.accueil.htm

Balacheff, N. & Gaudin, N. (2002). *Students conceptions: an introduction to a formal characterization.* Les cahiers du laboratoire Leibniz, n° 65. Grenoble: IMAG.

Bottino, R.M. & Chiappini, G. (2002). Advanced Technology and Learning Environments: Their Relationships Within the Arithmetic Problem-Solving Domain. In, L.D. English (Ed.), *Handbook of International Research in Mathematics Education*. Hillsdale, N.J.: Lawrence Erlbaum Associates .

Bottino, R.M. (2001). Advanced Learning Environments: Changed Vies and Future Perspectives. In, M. Ortega, J. Bravo (Eds), *Computers and Education Towards an Interconnected Society*, pp. 11-26. Dordrecht: Kluwer Academic Publishers.

Brousseau, G. (1997). *Theory of didactical situations in mathematics.* Dordrecht: Kluwer Academic Publishers.

Cerulli, M. (2004). *Introducing pupils to algebra as a theory: L'Algebrista as an instrument of semiotic mediation*. Doctoral thesis. University of Pisa.

Cerulli, M., Pedemonte, B., Robotti, E. (2005). An integrated perspective to approach technology in mathematics education. *Proceedings of CERME 4*, San Felix de Guixols, February 2005 (to appear).

Chevallard, Y. (1992). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, 12/1, 77-111.

Cobb, P. & Yackel, E. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics, *Journal for Research in Mathematics Education*, 27(4), 458–477.

Cole, M. & Engeström, Y. (1991). A cultural-historical approach to distributed cognition. In, G. Salomon (ed.), *Distributed cognition*, pp. 1-47. Cambridge, MA: Cambridge University Press.

Dershowitz, N, Jouannaud, J.P. (1989). Rewrite Systems. *Handbook of Theoretical Computer Science*, Vol B, Chap 15. North-Holland.

Di Sessa, A., Hoyles, C. & Noss, R. (1995). *Computers and exploratory learning*. NATO ASI Series. Berlin: Springer-Verlag.

Dreyfus, T., Artigue, M., Bartolini Bussi, M., Gray, E. (2005). Comparison of theoretical frames in mathematics education. *Proceedings of CERME 4*, San Felix de Guixols, February 2005 (to appear).

Engeström, Y (1991). Activity theory and individual and social transformation. *Activity theory* 7/8, 6-17.

Hall, E.T. (1981/1959). *The Silent Language*. NewYork: Anchor Press.

Halliday, M.A.K. (1978). *Language as Social Semiotic: The Social Interpretation of Language and Meaning*. London: Edward Arnold.

Halliday, M.A.K. & Hasan, R. (1989). *Language, Context and Text: Aspects of Language in a Social-Semiotic Perspective* (2nd ed.). Oxford: Oxford University Press.

ITD (Ed.) (2004). *Theoretical frameworks of reference*. www.itd.cnr.it/telma.

Kafai, Y. & Resnick, M. (Eds) (1996). *Constructionism in practice. Designing, thinking and learning in a digital world.* Hillsdale, N.J.: Lawrence Erlbaum Associates.

Kaptelinin, V. (1997). Activity Theory: Implications for Human-Computer Interaction In, B.A. Nardi (Ed.), *Context and consciousness*. Cambridge, MA: MIT Press.

Kynigos, C. & Argyris, M. (2004). Teacher beliefs and practices formed during an innovation with computer-based exploratory mathematics in the classroom. *Teachers and Teaching: theory and practice*, 10/3, 247-273.

Kynigos, C. (Ed.) (2004). *Intermediary report on learning contexts*. www.itd.cnr.it/telma

Kynigos, C., & Theodosopoulou, V. (2001). Synthesizing Personal, Interactionist and Social Norms Perspectives to Analyze Student Communication in a Computer - Based Mathematical Activity in the Classroom. *Journal of Classroom Interaction*, 36/2, 63-73.

Kynigos, C. (2004). A Black and White Box Approach to User Empowerment with Component Computing. *Interactive Learning Environments*, 12/1–2, 27–71.

Kutti, K. (1996). Activity theory as potential framework for human computer interaction research. In, B.A. Nardi (ed.), *Context and consciousness*, pp. 17-44. Cambridge, MA: MIT Press.

Lagrange, J.B., Artigue, M., Laborde, C., Trouche, T. (2003). Technology and Mathematics Education : A Multidimensional Study of the Evolution of Research and Innovation. In, A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick and F.K.S. Leung (Eds.) *Second International Handbook of Mathematics Education*, pp. 239-271. Dordrecht: Kluwer Academic Publishers.

Lave, J. (1988). *Cognition into practice.* Cambridge, MA: Cambridge University Press.

Lerman, S. (2000). The social turn in mathematics education research. In, J. Boaler (Ed.) *Multiple perspectives on mathematics teaching and learning*, pp. 19–44. Westport, CT: Ablex.

Margolinas, C. (2001). Situations, milieux, connaissances – Analyse de l'activité du professeur. In, J.L. Dorier, M. Artaud, M. Artigue, R. Berthelot, R. Floris (Eds), *Actes de la 11ème Ecole d'Eté de Didactique des Mathématiques*, pp. 141-156. Grenoble: La Pensée Sauvage.

Mariotti, M. A. (2002). Influences of technologies advances in students' math learning. In, L.D. English (Ed.), *Handbook of International Research in Mathematics Education*, pp. 757-786. Hillsdale, NJ: Lawrence Erlbaum Associates.

Mariotti, M.A. (2001). Introduction to proof: the mediation of a dynamic software environment. *Educational Studies in Mathematics*, 44, 25-53..

Mercer, N. (1993). Computer based activities in classroom contexts. In, P. Scrimshaw (ed.) *Language, classrooms and computers*. London: Routledge.

Morgan, C. (2002). What does Social Semiotics have to offer Mathematics education research. In, A.D. Cockbrun & E. Nardi (Eds), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*. Norwich: East Anglia University Press.

Morgan, C. (Ed.) (2004). *Intermediary report on representations*. www.itd.cnr.it/telma

Nardi, E. (1996). *The novice mathematician's encounter with mathematical abstraction. Tensions in concept –image formation and formalisation*. Doctoral thesis. University of Oxford.

Nicaud, J.F. (1994). Building ITSs to be used: Lessons Learned from the APLUSIX Project. In, R. Lewis, P. Mendelshon (Eds), *Lessons From Learning*, pp. 181-198. IFIP, North Holland.

Noss, R., Hoyles, C. (1996). *Windows on mathematical meanings, learning cultures and computers.* Dordrecht: Kluwer Academic Publishers.

Noss, R., Healy, L., Hoyles, C. (1997). The construction of mathematical meanings: connecting the visual with the symbolic. *Educational Studies in mathematics*, 33, 203-233.

Papert, S. (1980). *Mindstorms: children, computers and powerful ideas*. Harvester Press.

Pea, R.D. (1987). Cognitive technologies for mathematics education. In, A. H. Schoenfeld (ed.), *Cognitive science and mathematics education*, pp. 89-12. Hillsdale, NJ: Lawrence Erlbaum Associates.

Pernin, J.P. (2005). Mieux articuler connaissances, artefacts informatiques et environnements d'apprentissage: vers un modèle d'ingénierie des dispositifs centré sur les scénarios. *Contribution to the Symposium REF*, Montpellier, September 2005 (to appear).

Rabardel, P. (1995). *Les hommes & les technologies. Approche cognitive des instruments contemporains.* Paris: A. Colin.

Robert, A. (2001). Les recherches sur les pratiques des enseignants et les contraintes de l'exercice du métier d'enseignant. *Recherches en Didactique des Mathématiques*, 21/1-2, 57-80.

Salomon, G. (1996). Computers as a trigger for change. In, S. Vosniadou, E. De Corte, R. Glaser & H. Mandl (Eds.), *International perspectives on the design of technology-supported learning environments*, pp. 363-377. Hillsdale, NJ: Lawrence Erlbaum Associates.

Sierpinska, A. (1994). *Understanding in Mathematics*. London: The Falmer Press.

Sierpinska, A., Lerman, S. (1996). Epistemologies of mathematics and mathematics education. In, A. Bishop & al. (Eds), *Handbook of Research* *in Mathematics Education*, pp. 827–876. Dordrecht: Kluwer Academic Publishers.

Tchounikine, P. (2004). *Platon-1. Quelques dimensions pour l'analyse des travaux de recherche en conception d'EIAH*. http://archiveseiah.org/Référence HAL:ccsd-00002999, version 1.

Tricot, A., Plégat-Soutjis, F., Camps, J.-F., Amiel, A., Lutz, G., & Morcillo, A. (2003). Utilité, utilisabilité, acceptabilité : interpréter les relations entre trois dimensions de l'évaluation des EIAH. In, C. Desmoulins, P. Marquet & D. Bouhineau (Eds). *Environnements informatiques pour l'apprentissage humain*, pp. 391-402. Paris: ATIEF / INRP.

Vergnaud, G. (1991). La théorie des champs conceptuels. *Recherches en didactique des mathématiques*, 10(2/3), 133-169.

Vermersch, P. & Maurel, M. (Eds) (1997). *Pratiques de l'entretien d'explicitation*. Paris: ESF.

Vygotsky, L.S. (1978). *Mind in Society. The Development of Higher Psychological Processes*. Harvard University Press.

Vygotsky, L.S. (1962). *Thought and language*. Cambridge, MA: MIT Press.

[i] www.itd.cnr.it/telma

[ii]A cultural vision inspired by Hall (1981/1959) has been used in the mathematics education area by researchers such as Sierpinska (Sierpinska, 1994) and Nardi (1996). Hall recognizes three types of consciousness, three types of emotional relations to things: the formal, the informal, and the technical. In the context of mathematical culture, the formal level corresponds to beliefs around what is mathematics about, what are the legitimate tools and methods for mathematical work, etc; the informal level corresponds to schemes of action and thought, unspoken ways of doing things or thinking which result from experience and practice; and the technical level corresponds to the explicit part of knowledge, to the techniques and theories. This can be adapted to the research domain, attracting our attention to the role played by the formal and informal parts of our research culture in our research activities.

[iii] It would certainly be clearer to use the word instrument for the whole, and the word artefact according to Rabardel (Rabardel, 1995) or tool for denoting the concrete or non concrete object.

[iv] Approche Interdisciplinaire pour les Dispositifs informatisés d'Apprentissage. *www.math-info.univ-paris5.fr/**AIDA***

[v] We refer here to the distinction established in (Tricot, 2003) between three dimensions for ergonomic analysis : « utilisability » evaluates the tool according to its accessibility and facility of use, « utility » evaluates if the tool really does what it is supposed to do, « acceptability » evaluates its acceptability by its prospective users (persons or institutions).

[vi] Other agents can be the other students, the teacher, tutors as well as virtual agents such as the companions implemented in some ICT tools.