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ABSTRACT. *This paper focuses on an activity in which students explore sequences through a game, using ToonTalk programming and a web-based collaboration system. Our analytical framework combines theory of communities of practice with domain epistemology. We note three factors which influence the length and quality of interactions: facilitation, reciprocation and audience-awareness.*

Introduction

This paper tells the story of an experiment to design a mathematical community of practice, in the course of the *WebLabs* Project, a 3 year EU-funded educational research project oriented towards finding new ways of representing and expressing mathematical and scientific knowledge in communities of young learners. Our work focuses on the iterative design of exploratory activities in domains such as numeric sequences, cardinality, probabilistic thinking, fundamental kinematics, and ecological systems. In this paper, we will focus on an activity called *Guess my Robot*, which is aimed at advancing students' understanding of number sequences. We use that activity to explore the following question:

> *What are the factors that sustain interaction in a mathematical activity over a web-based collaboration medium?*

Our analysis is informed by the notion of 'community of practice' as it is used within the situated approach to learning (Lave and Wenger, 1991; Wenger 1998). The insights we gain from this analysis are fed into the next iteration of the activity design. Thus, we have built on our initial observations of communities to actively cultivate their existence.

Wenger proposes three dimensions of practice as the property of a community:

- Mutual engagement: a sense of "working together". Sharing ideas and artefacts, with a common commitment to the interactions between members of the community.
- Joint enterprise: having some object as an agreed common goal, defined by the participants in the very process of pursuing it, not just a stated agenda but something that creates among participants relations of mutual accountability; that become an integral part of the practice.

- Shared repertoire: agreed resources for negotiating meanings. This includes routines, words, tools, procedures, stories, gestures, symbols, and so on. Artefacts that the community has produced or adapted in the course of its existence and have become part of its practice. The repertoire combines both reificative and participative aspects. It includes the discourse members use to create meaningful statements about the world as well as the styles in which they express their forms of membership and their identities as members.

To these we add an epistemological dimension, in that we intend to encourage the formation of *mathematical* communities. That is, we are trying to generate communities of practice – both physically and virtually – in which there are agreed socio-mathematical norms, where it is natural to make conjectures, test hypotheses, offer counter-examples and so on. By restricting our attention to a specific domain of mathematical activity, we commit ourselves to make specific and concrete claims. Our focus on design provides us with a unique opportunity to go beyond explanatory observations. We can verify our claims by changing the activity system and monitoring predicted change.

WebLabs, ToonTalk, WebReports and the Guess my Robot game

WebLabs utilizes two main media for its activities: *ToonTalk* (a programming environment) and *WebReports* (a web-based collaboration system). We see programming as playing a key role in individual and group learning. Students explore and test their conceptions of the phenomena through programming. Furthermore, by sharing programmed models, they can communicate ideas in a concrete yet rigorous form. We are programming with *ToonTalk*[1] (Kahn, 1996; 1999; Mor et al., 2004) a language used in the past with younger children to construct video games (Hoyles, Noss & Adamson, 2002). *ToonTalk* is a computer game, programming environment and programming language in one. In *ToonTalk* programs take the form of animated cartoon robots. Programming is done by training these robots: leading them through the task they are meant to perform. After training, programs are generalised by "erasing" superfluous detail from robots' "minds".

The individual and collaborative facets of learning are intertwined at all stages of our activities. The *WebReports*[2] system was set up to support both. The primary aim of this system is to allow learners to reflect on each others work by sharing working models of their ideas. The "atomic unit" of content in the system is a web report: a document containing formatted text, multi-media objects and most importantly – ToonTalk models. Reports are edited using a visual editor. Students can grab any model constructed in their *ToonTalk* environment, and copy it instantaneously into their report. These models are embedded in the report as images, which link to the actual code object. When clicked, they automatically open in the reader's ToonTalk environment – which could be in another classroom or another country. The reader can then manipulate the object, modify it, and even respond with a comment that may include her own model. This last point is crucial: rather than simply discussing

[1] http://www.*ToonTalk*.com

[2] http://www.weblabs.org.uk/wlplone/.

what each other *thinks*, students can share what they have *built* and *rebuild* each others' attempts to model any given task or object.

Our activity design methodology exploits the affordances of the system. The initial discussion of a phenomenon can lead to the group's publishing a report on their observations, conjectures, and suggested path of inquiry. Finally, when a task or activity is completed, a concluding report will be published by either individuals or the group, to share conclusions with remote peers.

One of the experiments we have conducted in the course of the WebLabs project was a game called Guess my Robot. The activity we designed was based on the "Guess my rule" game, an activity well-known to many teachers and researchers as a way of encouraging students to discuss and compare the formulation of rules, and in particular the equivalence (or not) of their algebraic symbolism. It has also been employed in the context of Logo and spreadsheets (c.f. Healy & Sutherland, 1990). In its classical form, it has been used as an introduction to functions and to formal algebraic notation. As Carraher and Earnest (2003) have recently reported, even children in younger grades enjoy participating in this game, and can be drawn into a discussion of algebraic nature through using it.

We first experimented with the Guess my Robot activity in 2002/3 (Mor & Sendova, 2003). Our experience from this pilot informed both the design of the activity and of the collaboration system. In 2003/4 we expanded the experiment, with significantly greater response. This iteration included 33 students from 6 sites (in different European countries). There are several differences between our version of the game and other variations. Most notable is the media by which it is conducted, and the specific rules of game inspired by those. In our game, *proposers (*students) invent a rule for a number sequence and model it as a ToonTalk *robot* (procedure) that generates that sequence. They then collect the first few terms of its output in a ToonTalk *box* and embed it in a web report. *Responders* can click on the image of the box, and explore its contents in their own ToonTalk environment. They use a variety of tools to uncover the rule of the sequence: ToonTalk programming, Excel and (even!) paper and pencil. Once they succeed, they respond to the challenge by posting a comment on the report, which includes a robot they created for generating the same sequence.

Figure 1: Rita's Guess my Robot page

Figure 1 shows an example of such a challenge. It was posted by Rita[3], a 14 year old girl from Portugal. This example will accompany us throughout this paper. Rita's challenge provoked several different solutions, which led to long threads of interaction, some of which included fairly sophisticated mathematical arguments. Not all of our data is so impressive: overall, 45 challenges and 33 responses were posted. However, only 17 of the challenges received any response at all. A lot can be said about those challenges and responses – their mathematical structure and its relation to the tools used; the forms of expression which evolved through the game; how students construct their challenges, and how they select a challenge to respond to; the evidence all these present on questions of meta-cognitive skills and practices and so on.

Data and methods

The present dataset encompasses 33 students from 6 sites, 15 girls and 18 boys, ages 10 (2), 11 (10), 12 (16), 13 (2) and 14 (3). Challenges were posted between 26th December 2003 and 5th May 2004. The last response was submitted on 28th May 2004. Overall, 45 challenges and 33 responses were posted. Only 17 of the challenges received a response (obviously, some received more than one – a maximum of three per challenge). However, there are 114 comments altogether, up to 30 per a single report (3rd quartile at 3.25). The subject group is highly diverse. Each site had its own characteristics in terms of student selection, class setting, age, ethnic background, gender, and teacher-student ratio.

From a methodological point of view, one of the advantages of using a web-based collaborative system is that it is a self-documenting medium. All the challenges and responses posted by students, as well as any verbal comments, are archived and dated on the system. This data is abundant and easily accessible. Yet at the same time

[3] We use the aliases, or "handles" children chose for themselves in the web reports system. With the system's access restrictions in mind, we can use these as anonymized identifiers.

it is shallow: it does not record the classroom interactions or the problem-solving strategies used by the students. Analyzing this data cannot provide answers about personal and group learning trajectories, but it can point to interesting questions, such as:

- Students developed an ability to flow between different representations of the same sequence. In what ways does this ability affect their understanding of the mathematical objects they manipulate and the methods they use?
- The structure of the game requires participants to make conjectures, model them by programming, and test them. Does this facet of the activity influence students' mathematical argumentation?
- We identified several canonical structures of sequences which appeared in many challenges and in different sites. These structures are notably different then those taught in standard curricula. What are the epistemological sources of this difference, and what are their implications?

These questions are then explored by looking at field notes, session recordings and interviews across sites. In this paper we wish to focus on one theme, the issue of sustaining interaction in a mathematical game, within a web-based collaborative system. The next section elaborates this question.

Sustaining mathematical interaction

It is clear that sustaining the kind of interaction we seek is strongly contingent on the domain, the activity structures, and, of course, the tools that we offer to students. Nevertheless, as in any learning environment, the epistemological, cultural and social factors are intertwined. Thus, our answers cannot be detached from social and cultural considerations.

Asking how to sustain interaction implicitly suggests that it is a positive force. Yet this is itself a claim that needs to be scrutinized. In the case of Rita's challenge, the first responses were bare robots. As the interaction developed (in fact, in several concurrent threads) students went deeper and deeper into the questions that emerged from the situation: equivalence of models, solution strategies and even notions of proof. Participants shifted from the competitive and somewhat technical base level of the game to a collaborative effort of understanding the mathematical structure of their models, and sharing of analytical tools.

Assuming we accept sustained interaction as a desirable phenomenon, we need to look closely at the cases were it occurs and try to identify their unique characteristics. We should obviously pay closest attention to cases were the interaction is distinguished not only by quantity but also by quality. That is, quality of the mathematical and meta-mathematical discussion exhibited in the interaction. There are 3 main themes that have emerged from our preliminary observations: facilitation, reciprocation and audience-awareness.

Facilitation

Our first conjecture regards the role of the facilitator. As Wenger et al. (2002) note, "*Alive communities, whether planned or spontaneous, have a 'coordinator' who organizes events and connects community*". We assert that this role of coordination, or facilitation in our terms, is critical in maintaining the dynamics of the game. Facilitation takes on three forms:

- Technical: providing technical apprenticeship on how to use the system, e.g. how to post a response; pointing teachers and students to interesting postings.
- Pedagogical: setting new challenges to participants; noting the mathematical or computational aspect of postings to teachers and students.
- Sociomathematical: shifting the conversation towards mathematical content. In the terminology of Yackel & Cobb (1995), establishing the sociomathematical norms of the game.

At first, the Bulgarian students posted their response in a separate report. Yishay copied the text and the robots from their reports and posted them as comments on Rita's challenge. He then e-mailed the teachers at both sites about this. Obviously, this is not a very interesting event to report. Nevertheless, none of the following discussions about sophisticated mathematical ideas would have occurred without it.

As an example of promoting sociomathematical norms, consider the following comment posted by the London researchers:

> This is a question from the London team (Richard, Celia, Ken, Yishay and Gordon) to all three of you:
>
> We think your robots will generate the same sequence for ever, but how can we be sure?

This question provoked students in both sides to think about the question of equivalence. The Bulgarians approached this question by working it out algebraically in a group. Rita considered this option, but thought that the rules of the game restricted her to using ToonTalk. Her solution was to construct a robot that compares two sequences by subtracting respective terms. She explains:

> Clearly that this is not a prove of that robot produces the same sequence, that is only one conjecture, or either, I have 99% of sure that they are equal, but still did not can to get a demonstration.

One of the responses to Rita's difference robot is an example of a pedagogical intervention. Gordon comments:

> Wow - this is really great work! Did you know that you could actually create other sequences using the difference robot that you built? I.e. if the two robots you send off in the trucks don't generate the same sequence, then your difference robot will generate a sequence of non-zero numbers. Try it!

Gordon suggests a new challenge, based on the work that Rita had published. Unfortunately, at this point we have to report a lack of success. Rita responded politely, but did not pick up the challenge. Her teacher's field notes reveal an explanation: she answered the comment, and was disappointed not to receive a response from Gordon. It was not a lack of interest in the mathematical problem, but rather a suspicion that Gordon would not maintain the interaction on his side. We will return to this important observation later, when we mention the issue of presence.

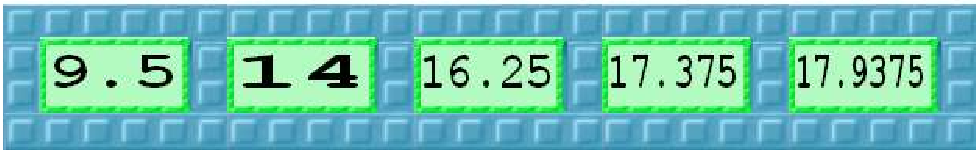
Using a web-based medium eliminates constraints of organizational structure. An expert in London or Portugal can facilitate activity in a classroom in Cyprus. The WebReports system includes several features which aid facilitation. For instance, challenges are listed automatically, with the number of comments they received. The facilitator can identify challenges which have not been responded to, and use the system's messaging facility to invoke other participant's awareness to them. Whenever the facilitator identifies a common technical or conceptual problem, she can publish a tutorial which addresses it.

Reciprocation

A second theme we identify is reciprocation. Under some circumstances, students feel a stronger obligation to reply than others. These circumstances may have a social element, for instance the sense of obligation is stronger when a comment is posted by a group of students or by a teacher. On the other hand, a very strong element in reciprocation is a socio-mathematical factor: participants sense they should "give something in return" for a positive experience, and solving a tough challenge is seen as such. Thus, participants' tendency to respond rises with the difficulty of the challenge. This conjecture addresses not only the frequency of responses, but also their quality: when the challenge was gratifying, students respond with more then their solution, adding unexpected levels of mathematical discourse to the interaction.

When Nasko posts his response to Rita's challenge, he adds:

> Here is also a sequence generated by the same robot. Two questions:
>
> 1. What was the input of my robot?
>
> 2. Can your robot generate it?



Nasko's response dissects the *process* of generating the sequence from its *initial conditions*, giving rise to the idea that the same process can produce different mathematical objects.

Rita responds in two stages. First, she reciprocates on the social level – congratulating Nasko on his response, and sharing her original model with him. She explains to her teacher that she should respond immediately so as not to discourage him. Only then does she set on solving his challenge. After she does that, she reciprocates on a domain knowledge level, by posting her solutions.

The flip side of this phenomenon is that students do not respond to challenges they see as uninteresting. Sometimes, a student might pick up a simple challenge as a "drilling challenge", but will not invest in posting her solution. At the end of the activity, we asked students to publish a concluding reflective report. When asked about the responses to her challenge, one girl responded:

> I don't receive any comments to my sequence, because is to easy...

Reciprocation is so natural in classroom practice that it goes unnoticed: a teacher acknowledges a student's remark; students support each other's claims. In a web-based environment it raises tensions which we need to accommodate. Teachers need to actively seek students' contributions and react to them, less the students feel unnoticed. Other issues arise from the need to adjust to asynchronous communication: at the beginning of one session, Rita posted a comment and then sat back, waiting for a reply, growing frustrated by the minute. Her teacher had to explain that although she could see several members of the community on-line, they might be occupied with other activities and unaware of her comment.

On the positive side, streamlining the ToonTalk objects into the text of the reports had the effect of enriching students' interactions. When Nasko posted his robot as a response to Rita's challenge, she reciprocated by posting hers. This gave rise to the question of comparing the robots and asserting their equivalence. Since robots, as coded objects, are by nature formal structures, the discussion took a much more formal tone than may have been the case with bare text.

Audience-awareness

Our last conjecture is perhaps the most socially-oriented. We find that two characteristics of a participant provoke response to her contributions: cordiality and presence. The first is almost trivial – participants respond more eagerly to friendly, inviting comments. The second is accentuated by the medium we chose, and in a way related to the issue of reciprocity. We find that participants prefer to interact with peers who project a strong presence. (e.g. appear on the "active users" list, post frequent comments, have a rich home page). Our conjecture is that this stems from the fact that participants are in fact interested in sustained interactions, and thus prefer to communicate with peers (or researchers) from whom they expect a higher probability of response. This entails immediate implications for us: participants are set back by one-off comments, and researchers should refrain from commenting if they do not intend to participate in subsequent discussion.

An example of this idea has been mentioned above: Rita did not attempt to solve Gordon's challenge because she suspected he might not be available to appreciate her response.

On the positive side, a team of Cypriot students replied to Rita's challenge nearly a month after the previous interactions. Because they identified themselves as a team, Rita felt a stronger commitment to her audience. She felt obliged to reply to the Cypriots, and to do so thoughtfully. The Cypriots volunteer an explanation of their solution strategy:

> 1. We copied Rita's numbers in Excel, to be easier to find relations between the numbers and especially the differences.
>
> 2. We found the differences between the numbers on that sequence.

3. We noticed that differences between numbers could be calculated if we multiply every one difference by 4.

4. So, we decided that we could work with formula 4* number.

5. To get Rita's sequence, we had to add 8 to the previous formula. The final formula is 4*number+8

Best

Cyprus Mathematics WebLabs Team

And Rita responds by taking the role of the facilitator, and elevating the discussion:

I can prove that my sequence and your sequence are equal with the process of algebraic representation used by Sofia group.

Rita's sequence:

A1 = 2

An+1 = (An + 2) x 4, but if I using the distributive property of the multiplication relatively to the addition I can write that:

A1 = 2

An+1 = An x 4 + 8

That is the algebraic representation of the Cyprus's sequence. Then I can prove that two sequences are equal.

Conclusions

In this paper we have explored the question of sustaining interaction in a mathematical activity over a web-based collaboration medium. Our approach attempts to interleave the theoretical framework of communities of practice with epistemological observations arising from the specific knowledge domain of number sequences. As a case study, we have chosen one of our experiments involving a game called *Guess my Robot*. Our analysis suggests several factors which contribute to the extent and to the quality of interactions: facilitation, reciprocation and audience-awareness. Supporting these elements has guided our design of the webreports system. Nevertheless, along with its potentials the technology raises challenges – which need to be addressed by adjusting patterns of behaviour as well as social norms. The fundamental elements of a community of practice are reflected both in our analysis and in the design of the tools, the rules and the roles in our activities.

Mutual engagement, in the sense of sharing and discussing artefacts, is afforded by the features built into the WebReports system; its support of joint and individual authoring of documents, the ease of commenting on others' document, and most importantly – the ability to include models of ideas as manipulable objects in these documents. The notions of facilitation and reciprocation elaborate on the idea of mutual engagement. Implicit rules of engagement emerge by which, for example, harder challenges are more esteemed and provoke richer responses.

A sense of joint enterprise is valuable in motivating students to engage in the activity. This motivation is related to participants' audience-awareness; a factor that is easy to neglect in traditional environments, but takes prominence in a web-based environment, where the communication channels are thin. As the accepted value of the enterprise rises, in terms of its mathematical richness, so does the level of collaboration.

The concept of shared repertoire is related to that of sociomathematical norms, but also the domain-specific questions, such as the implicit agreement on what constitutes a hard challenge and the positive value of one. Using programming (specifically ToonTalk) as a taken-as-shared resource enriches the repertoire with a language that is both rigorous and expressive. As students master the multiple facets of their repertoire, the boundaries between the verbal and computational languages they use are blurred. Their argumentation is shaped by the tools, while at the same time they shape the tools to express their arguments.

Synergising distinct paradigms is always a challenging task. In our case, we still see more questions than answers before us, but these questions are enough to make the effort worthwhile.

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